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ERA treatment of asset pricing models

Dr. Tom Hird

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COMPETITION ECONOMISTS GROUP

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1 Introduction

1. My name is Thomas Nicholas Hird and I am a founding director of CEG Asia Pacific (**CEG**) and head of its Melbourne office. I have a Ph.D. in Economics from Monash University and am an Honorary Fellow of its economics faculty. I have more than 20 years of experience in the economic analysis of markets and in the provision of expert advice in regulatory, litigation and policy contexts. I have provided expert testimony before courts and in numerous regulatory forums.

2. I have been asked by DBP to provide an expert assessment of the ERA's treatment of asset pricing models in its draft decision for ATCO.[1] IN particular, I have been asked to provide my views on the ERA's decision to give no weight to estimates of the cost of equity derived from application of the Fama French three factor model and the Empirical CAPM.

1.1 Terms of reference

3. My terms of reference for this report are set out below.

> *In its Rate of Return Guidelines published 16 December 2013 (Guidelines), the ERA sought to undertake a review of several different asset pricing models; the SL-CAPM, the Black CAPM, the Inter-temporal CAPM, Arbitrage Pricing Models, the Fama-French Model, Dividend Discount Models, Residual Income Models, Market Risk Premium Approaches and the Build-Up Method. It assessed these according to a set of criteria which it suggested were associated with the Allowed Rate of Return Objective in NGR 87(3) (ARORO), rather than the ARORO itself, and found that all except the SL-CAPM were not fit for purpose.*
>
> *In its submission lodged with the ERA in April 2104 ERA as part of its access arrangement approvals process, ATCO Gas Australia examined the assessments made by the ERA, and disagreed with it on a number of points. The ERA responded to these disagreements in its recent Draft Decision on Proposed Revisions to the Access Arrangement for the Mid-West and South-West Gas Distribution System, dated 14 October 2014 (ATCO Draft Decision).*
>
> *DBP is seeking an opinion which takes into account these three documents (the Guidelines, the submission by ATCO Gas and the ATCO Draft Decision), and the literature in general, to understand whether, in accordance with Rule 87(5) of the National Gas Rules, different models*

[1] When I refer to the "ATCO draft decision" I mean ERA, Draft Decision on Proposed Revisions to the Access Arrangement for the Mid-West and South-West Gas Distribution System (2014).

might be considered relevant, and thus capable of providing information towards developing an estimate of the rate of return which could be held to meet the ARORO. This would involve an assessment of the arguments made in each of the three documents, and a final viewpoint in respect of each model.

Please prepare a report with addresses (as necessary) the following matters:

i. *A response to the ERA's views on the merits of models other than the Sharpe Lintner CAPM in both the Guidelines and also its ATCO Draft Decision.*

ii. *Identification of any other issues not covered in the three documents above which might be important for ascertaining the relevance of a particular asset-pricing model.*

iii. *A discussion of the relevance of bias and variance in the estimates coming from different models and how, if at all, this is relevant to the weight that should be given to different models.*

1.2 Structure of this report

4. We address the above issues in the remainder of this report, which is structured as follows:

- **Section 2** describes the ERA's reasoning.
- **Section 3** provides my critique of that reasoning.
- **Section 4** describes the trade-off between precision and bias and how, in theory, the ERA should assess the weight given to each model.
- **Appendix A** provides an extract from a report by professor Grundy surveying the relevant empirical literature.

6. I acknowledge that I have read, understood and complied with the Federal Court of Australia's *Practice Note CM 7, Expert Witnesses in Proceedings in the Federal Court of Australia*. I declare that I have made all the inquiries that I believe are desirable and appropriate and that no matters of significance that I regard as relevant have, to my knowledge, been withheld.

Thomas Nicholas Hird

23 December 2014

2 ERA treatment of models other than SL CAPM

7. The ERA has determined that it will place sole reliance on the Sharpe Lintner CAPM at the expense of other models including the Fama French 3 factor model (FFM) and the Empirical CAPM.

8. The Commission first made this decision in its rate of return guideline process and has adhered to this in the ATCO draft decision. The following quote from the draft decision summarises the ERA's reasoning.

> *663. The Authority concluded that only the Sharpe Lintner CAPM model is relevant for informing the Authority's estimation of the prevailing return on equity for the regulated firm at the current time. The Authority considered that incorporating returns from other models would detract from the ability of the Authority to meet the allowed rate of return objective.*
>
> *664. However, the Authority determined that it would give weight to relevant outputs from the DGM when estimating the market risk premium (MRP), which is an input to the Sharpe Lintner CAPM.*
>
> *665. The Authority also noted the empirical evidence provided by the Black and Empirical CAPM models, pointing to potential bias in the estimates from the Sharpe Lintner CAPM, and noted that it would take this information into account when estimating the point estimate of the equity beta from within its estimated range.*
>
> *666. The Authority concluded that other models and approaches are not relevant within the Australian context, at the current time, without some new developments in terms of the theoretical foundations or in the empirical evidence. Generally, there are resulting shortcomings with regard to robustness in the Australian context. On this basis, the Authority considered that these other models are not 'fit for purpose' or able to be 'implemented in accordance with best practice'.*

9. I describe the ERA's views in relation to the Empirical CAPM and FFM in each of the subsequent sections.

2.1 The Empirical CAPM

10. When I use the term Empirical CAPM, I am referring to an implementation of the CAPM where the return on a zero beta asset is set at a value that is above the risk free rate and where the magnitude of this 'zero beta premium' is determined by reference

to empirical studies of the relationship between returns and beta. This is consistent with the terminology used by the ERA in its explanatory statement to the rate of return guideline:

> *The Black CAPM was developed as a response to the ex post empirical assessment of the performance of the Sharpe-Lintner CAPM, and its resulting perceived shortcomings, particularly the 'low beta' bias referred to above at paragraph 23. The Black CAPM belongs to the Empirical CAPM family of models, which adjust the parameters of the Sharpe-Lintner CAPM to align better with the ex post outcomes that are observed.*[2]

11. I refer to the Empirical CAPM rather than the Black CAPM in order to be clear that I am not only referring to the theoretical conclusions of Black (1972)[3] but also the empirical work in Black Jensen Scholes (1972)[4] and many other similar studies. The ERA's terminology tends to use the term "Black CAPM" to encompass what I am referring to as the Empirical CAPM.[5]

12. In relation to the Empirical CAPM the ERA states:

> *693. Second, the Authority does not accept, as NERA does, that the Sharpe Lintner beta estimates do not reflect stock returns. The Authority rejected the use of the Black CAPM in the Rate of Return Guidelines, on the basis that its empirical performance was unreliable. The Authority considers these issues in more detail at paragraphs 734 to 757.*

13. While the ERA references paragraphs 734 to 757 as providing elaboration, the actual paragraphs that are relevant are only 754 and 755.

> *754. With respect to the Black CAPM, the Authority rejects SFG's assertion that this implies an equity beta of 1, based on the analysis conducted by NERA. First, the Authority rejected the use of the Black CAPM in the Rate of Return Guidelines,* ***on the basis that its empirical performance was unreliable****. Second, the Authority noted in the Rate of Return Guidelines that:*
>
> > *… the Authority intends to account for empirical evidence relating to potential bias in the estimates of the equity beta that are used in*

[2] ERA, Rate of Return Guidelines, Explanatory Statement, Appendix 8. p.64

[3] Fischer Black, Capital Market Equilibrium with Restricted Borrowing The Journal of Business , Vol. 45, No. 3. (Jul., 1972), pp. 444-45.

[4] Fischer Black, Myron Scholes, & Micheal Jensen, "The Capital-Asset Pricing Model: Some empirical tests", in Jensen, editor, Studies in the Theory of Capital Markets (1972).

[5] The ERA uses the term "Empirical CAPM" only twice at paragraphs 665 and 754 but elsewhere uses the term "Black CAPM" to encompass what I term the "Empirical CAPM".

> *applying the Sharpe Linter CAPM. The Authority considers that such an approach would account for much of the evidence supporting the use of the Empirical and Black CAPM models.*
>
> *755. The Authority considers that the Black CAPM is only useful to the extent that it suggests a downward bias in the return on equity generated by the Sharp Linter CAPM for firms with an equity beta less than 1. The Authority is of the view that it is difficult to quantify the extent of any downward bias.* [Emphasis added.]

14. In the above passage the ERA:

    - restates a position from the rate of return guideline process that the Black (Empirical) CAPM's empirical performance is unreliable; and
    - states a view that the Black CAPM is only useful "to the extent" that it suggests a downward bias in the return on equity generated by the Sharp Linter CAPM for firms with an equity beta less than 1.

15. Dealing with the asserted unreliability one must go to the guidelines explanatory statement to understand the basis of this decision. The relevant section of that decision begins at paragraph 41 of Appendix 8 and ends at paragraph 50. It is somewhat difficult to discern from this discussion the basis for the ERA's conclusion that the empirical performance of the Black (Empirical) CAPM is unreliable. The ERA does state that it had previously expressed this view.

> *43. The Authority considered NERA's proposal provided by Dampier to Bunbury Pipeline (DBP) at length as part of its determination for its decision on the Dampier to Bunbury Natural Gas Pipeline access arrangement.[108] The Authority concluded that the Black CAPM was not widely used in Australia, and did not produce reliable estimates within an Australian context.*
>
> *108 Economic Regulation Authority 2011, Final Decision on Proposed Revisions to the Access Arrangement for the Dampier to Bunbury Natural Gas Pipeline, www.erawa.com.au, p. 156*

16. In order to understand the basis of this claim one must go back further to page 156 of the ERA's 2011 decision for DBNGP (as per ERA footnote 108 reproduced above). When I do this I find no explanation of why the ERA believes that Black (Empirical) CAPM is unreliable. What I do find is a restatement of a position in the draft decision that the Black (Empirical) CAPM is not a "well accepted model". When I turn to the Draft Decision I find the following passage:[6]

[6] ERA, Draft Decision on Proposed Revisions to the Access Arrangement for the Dampier to Bunbury Natural Gas Pipeline, March 2011, p.114

> *378. The Authority considers that four academic papers and one working paper do not constitute a significant body of evidence. A report prepared by NERA on the issue does not provide an explanation of how these four papers were selected, or even a justification of why these particular papers represent the opinion of a sufficient cross section of the academic literature participant class. In addition, four out of five academic papers presented were for the US capital market, whereas the relevant regulatory framework is concerned with Australian capital markets. As such, the Authority is of the view that the evidence presented does not reflect prevailing market conditions in which the reference services are provided to meet the requirements of rule 87(1) of the NGR.*
>
> *379. In addition, for regulatory certainty, the Authority rejects the use of the estimate of the zero-beta premium from Lajbcygier and Wheatley (2009) paper as it is only a working paper from Monash University and the paper has not yet been through the rigorous process generally required of a formal publication. As such, the findings should not be considered as being as reliable as findings from other papers in well known academic journals. The Authority is not aware of any empirical studies published in academic journals regarding the estimates of zero-beta premium for Australia or any commercial sources in which estimates of zero-beta premium are available.*
>
> *380. The Authority has not identified any evidence that the Black CAPM has been broadly applied by financial analysts and business practitioners in valuation or capital budgeting in Australia.*

17. Following the above threads, it appears that the ERA's conclusion in late 2014 that the Black (Empirical) CAPM is not reliable is based on the fact that the ERA did not find the set of papers submitted to it by NERA in 2010 to be sufficiently compelling.

18. The ERA does not appear to have, at any stage, researched what it believes is the best available evidence suggested by the Empirical CAPM. This is surprising given that it undertook that it would do so in the rate of return guideline process.[7]

> *25. The determination takes into account other relevant material, such as insights from the empirical performance of the Sharpe Lintner CAPM. The Authority considers that relevant empirical evidence supports a view that there is some downward bias in equity beta estimates that are less than one, and upward bias in equity beta estimates that are greater than one.*
>
> *26. Therefore, for the purposes of this indicative estimate, the Authority will assume a point estimate for the equity beta that is at the top end of the estimated range, at 0.7, so as to account for potential bias in the estimate.*

[7] ERA , Explanatory Statement for the Draft Rate of Return Guidelines, Appendix 30, p. 217

> *27. The Authority intends to undertake work to quantify the extent of this potential bias prior to its next decision. This work would then assist in informing the degree to which the Authority might adjust up the point estimate of the equity beta within the estimated range, so as to account for any potential beta bias.*

19. Notwithstanding saying that it would carry out this work prior to its next decision no such work has been carried out by the ERA. The ATCO draft decision does summarise the ERA's guidelines position including that it would undertake such work[8] but provides no explanation of why this has not occurred (or explicit acknowledgement that it has not occurred).

20. The explanatory statement to the guidelines does survey more recent discussion of the Empirical CAPM quoting from an exchange between McKenzie and Partington and NERA. However, the ERA does not conclude from this that the Empirical CAPM is unreliable *per se*. Rather, it states: [9]

> *47. This is a highly technical debate. Of most concern to the Authority is the inference from NERA's modelling that the zero beta portfolio has a return that is consistent with the MRP, leaving no room for variation in returns across stocks based on relative risk. The Authority considers that it is difficult to reconcile this result with either the theoretical underpinnings of portfolio theory, or actual observed outcomes in the market.*

21. Here the ERA is expressing the view that the results of a study by NERA are implausible because they leave "no room for variation in returns across stocks based on relative risk". Putting aside the fact that this is not actually what the NERA results suggest,[10] this is only one study. It is not clear why concluding that a single study results in implausible results is a basis to dismiss all studies as unreliable – especially in a context where the ERA has concluded that the weight of these studies do support a conclusion that low beta bias exists: [11]

> *The Authority considers that relevant empirical evidence supports a view that there is some downward bias in equity beta estimates that are less than one, and upward bias in equity beta estimates that are greater than one.*

22. As noted above, at paragraph 755 of the ATCO draft decision the ERA states that it

[8] See paragraph 743 on page 170 of the ERA's Draft Decision on Proposed Revisions to the Access Arrangement for the Mid-West and South-West Gas Distribution System (2014).

[9] ERA , Explanatory Statement for the Draft Rate of Return Guidelines, Appendix 8, p. 66 paragraph 47.

[10] The NERA results suggest that beta plays no role in determining returns on Australian stocks. This is not the same as saying that risk does not vary across stocks – just that beta does not provide a useful measure of risk.

[11] ERA, Explanatory Statement for the Draft Rate of Return Guidelines, Appendix 30, p. 217

> *…considers that the Black CAPM is only useful to the extent that it suggests a downward bias in the return on equity generated by the Sharp Linter CAPM for firms with an equity beta less than 1. The Authority is of the view that it is difficult to quantify the extent of any downward bias.*

23. This statement is difficult to interpret given that the ERA has already stated that it does believe that such a bias exists. The difficulty or otherwise of quantifying the bias can only be made clear if the ERA actually attempts to do so. Despite a declared intention to do so, no such attempt has yet been made by the ERA.

2.2 Fama French Model

24. In its ATCO Gas Draft Decision, the ERA repeats its position stated in the rate of return guideline process that the Fama French model is irrelevant.[12]

> *680. The Authority notes that the Fama French three factor model (**FFM**) has consistently been put forward by regulated businesses as a means to estimate the return on equity. However, in its previous regulatory decisions, the Authority concluded that there is no strong theoretical basis to support the inclusion of the two additional risk factors to estimate the rate of return on equity, as occurs in the FFM. This is because the FFM is dependent on empirical justification – that is, the systematic observance of the FFM risk premia. In contrast, given that the FFM risk premia are not systematically observed in the Australian market, there is no reasonable basis for the FFM to be applied in Australia.*
>
> *681. The Authority's recent analysis of the FFM in the context of the Australian market for equity, for this Draft Decision, shows that observed empirical evidence is not consistent with the FFM (refer to Appendix 4 of this draft decision).*

25. The ERA's reasoning here is clear and reflects a) concern that the model is not derived from theory; and b) work performed by the ERA suggests that the FFM does not explain historical Australian equity market returns.

[12] ERA, ATCO draft decision, p. 157.

3 Critique of ERA's rationale

26. I consider that the ERA's justification for giving no weight to the Empirical CAPM and the FFM models is not soundly based and its decision is unreasonable. The ERA's rationale is also internally inconsistent.

3.1 Empirical CAPM

27. As set out in section 2.1, the ERA's conclusion in the ATCO draft decision that the Empirical CAPM is unreliable is based on:

    - the views expressed in 3 paragraphs of a 2011 draft decision[13] which, in reality, did not find that the Empirical CAPM was unreliable but that:
        i. "four academic papers and one working paper do not constitute a significant body of evidence"; and
        ii. these papers, selected by NERA might not "represent the opinion of a sufficient cross section of the academic literature participant class"; and
        iii. "four out of five academic papers presented were for the US capital market, whereas the relevant regulatory framework is concerned with Australian capital markets"; and
        iv. the sole Australian paper "is only a working paper from Monash University and the paper has not yet been through the rigorous process generally required of a formal publication. As such, the findings should not be considered as being as reliable as findings from other papers in well known academic journals"; and/or
    - in relation to more recent evidence put to it "the inference from NERA's modelling that the zero beta portfolio has a return that is consistent with the MRP the fact that an empirical paper by NERA". [14]

28. I set out below the reasons why I do not find the above a reasonable basis for the ERA's conclusion that the Empirical CAPM is unreliable. However, before doing so, I note that an important context is that the ERA stated in its guideline process that: [15]

    - "*relevant empirical evidence supports a view that there is some downward bias in equity beta estimates that are less than one*"; and

[13] ERA, Draft Decision on Proposed Revisions to the Access Arrangement for the Dampier to Bunbury Natural Gas Pipeline, March 2011, p.114

[14] ERA , Explanatory Statement for the Draft Rate of Return Guidelines, Appendix 8, p. 66 paragraph 47.

[15] ERA , Explanatory Statement for the Draft Rate of Return Guidelines, Appendix 30, p. 217

- "*the Authority intends to undertake work to quantify the extent of this potential bias prior to its next decision. This work would then assist in informing the degree to which the Authority might adjust up the point estimate of the equity beta within the estimated range, so as to account for any potential beta bias*".

3.1.1 There is an abundance of evidence on the Empirical CAPM

29. The empirical literature on the extent of the bias for low beta stocks is well established. This is apparent from the submissions that the ERA had received prior to the ATCO draft decision. For example, a report by NERA[16] that the ERA cites in its explanatory statement states:

> *Mehrling (2005), for example, reports that:*
>
> > *'The very first [Wells Fargo] conference was held in August 1969 at the University of Rochester in New York State … The focus of the first Wells Fargo conference was on empirical tests of the CAPM … the most significant output of the first conference was the paper of Fischer Black, Michael Jensen, and Myron Scholes (BJS), titled "The Capital Asset Pricing Model: Some Empirical Tests," eventually published in 1972. … One important consequence of the BJS tests was to confirm earlier suggestions that low-beta stocks tend to have higher returns and high-beta stocks tend to have lower returns than the theory predicts.'* (Page 4)

30. That is, papers since Black Jensen Scholes (1972) have addressed this issue. Indeed, NERA references a study by CEG[17] that applied the Black Jensen Scholes (1972) methodology to Australian data and, unsurprisingly, came to the same conclusions as NERA, namely, that betas measured in the way the ERA measures them play very little, or no, role in explaining historical Australian equity returns. This result was consistent with the conclusions in international studies which the CEG report also surveyed which included a discussion of earlier studies such as Black Jensen Scholes (1972) and more recent surveys such as by Fama and French's 2004 paper entitled "The capital asset pricing model: Theory and evidence," published in the Journal of Economic Perspectives. We included the following quote from that paper.

> *"Fama and French (1992) also confirm the evidence (Reinganum, 1981; Stambaugh, 1982; Lakonishok and Shapiro, 1986) that the relation between average return and beta for common stocks is even flatter after the sample periods used in the early empirical work on the CAPM."*

[16]NERA Economic Consulting 2012, The Black CAPM: A report for APA Group, Envestra, Multinet & SP AusNet is cited on page 117 of the ERA's Explanatory Statement for the Draft Rate of Return Guidelines.

[17] CEG, Estimation of, and correction for, biases inherent in the Sharpe CAPM formula, September 2008.

31. Professor Bruce Grundy has also performed a recent review of the international literature. The sample of articles that Professor Grundy relied on for this survey were the 16 articles identified by the AER as articles that the AER alleged supported the use of the Sharpe Lintner CAPM.[18] Professor Grundy concluded that none of these 16 papers supported a conclusion that there was no low beta bias and that, of the papers that reported empirical results, all rejected such a conclusion. In four of these papers it was possible to derive, from the reported results, the zero beta premium[19] as a percentage of the market risk premium. This ranged from 77% to 24% with an average of 0.49%.

32. That is, of the studies relied on by the AER to support the use of the Sharpe Lintner CAPM all of them were consistent with the existence of low beta bias and the average implied zero beta premium was half of the market risk premium. I note that this result is consistent with recent estimates of the zero beta premium in Australia by SFG.[20]

33. In summary, the empirical literature that is relevant to low beta bias is very extensive and there is nothing preventing the ERA from surveying this literature and arriving at its own estimate of the zero beta premium (and, hence, the zero beta bias). Absent the ERA performing this study a conservative estimate would be that the zero beta premium is half of the market risk premium. This is conservative because it is based on the results reported in articles selected by the AER and used by it to support the reasonableness of the Sharpe Lintner CAPM (i.e., these articles can be expected to have a bias in favour of a low zero beta bias given the purpose to which the AER put them). It is also consistent with the lowest estimate of the zero beta premium using Australian data (SFG's estimate) – with the other two estimates (CEG and NERA) implying a higher zero beta premium.

3.1.2 The Empirical CAPM cannot be empirically unreliable

34. Purely as a matter of logic, the Empirical CAPM cannot be less empirically reliable than the Sharpe Lintner CAPM. Consistent with its name, the Empirical CAPM estimates the sensitivity of stock returns to equity beta based on the empirical data. As such, it gives the best estimate, based on the available data, of the role of beta in determining market returns.

[18] These papers are cited by the AER on pages 149 and 150 of AER, Final decision—Public, Jemena Gas Networks, Access arrangement proposal for the NSW gas networks, 1 July 2010 – 30 June 2015, June 2010.

[19] That is, the difference between the estimated return on zero beta assets and the risk free rate (where the latter is the prediction of the Sharpe Lintner CAPM)

[20] SFG, Cost of equity in the Black Capital Asset Pricing Model, May 2014. See paragraph 102 for source of the numbers in the calculation = (12.40-9.36)/(12.40-6.02)=3.34%/6.38%=0.48.

35. It is possible to conclude that the available data is imperfect and, therefore, the estimates derived from that data are imperfect. Certainly, this helps explain why not all studies arrive at the same estimate of the zero beta premium. However, all studies that I am aware of arrive at an estimate of the zero beta premium that is materially above zero. Professor Grundy states:

> *I know of no published study that has empirically tested the Sharpe Lintner CAPM and failed to reject the Sharpe Lintner CAPM.*

36. The empirically most reliable estimate is, therefore, that the zero beta premium is materially positive. We may not know with precision what that value is but we do know that it is not zero. Indeed, the ERA has acknowledged as much. Therefore, in my view, the ERA has acted unreasonably in failing to review the empirical evidence and to use that review to arrive at an estimate of the zero beta premium.

37. By contrast, the Sharpe Lintner CAPM estimate of the sensitivity of stock returns to equity beta is based on a set of stylised assumptions without regard to the empirical data. It is a purely theoretical model. It has been tested and its predictions are inconsistent with the evidence. Unlike the Empirical CAPM, it is possible to conclude that the Sharpe Lintner CAPM is empirically unreliable. This is a conclusion that would be unavoidable if the ERA were actually to carry out the review of the Empirical CAPM that it undertook to do in the guideline process.

3.1.3 Australian vs foreign studies

38. The ERA has argued in the DBNGP draft decision[21] that it required compelling evidence of a zero beta premium in Australia before adopting the Empirical CAPM. At the time of its decision it appears to believe that there was insufficient such evidence. The ERA appears to have relied on that decision to support a continuing conclusion in the ATCO draft decision[22] that Australian evidence was unreliable.

39. The first point to note in response to this is that international evidence is compelling. Wherever this has been tested, a zero beta premium has been found to exist. This is a settled conclusion in the finance literature. Even if there were no evidence specific to Australia the only reasonable *a priori* conclusion should be that the same result that has been found internationally will also be true in Australia. That is, the ERA should require evidence that the same relationship does not apply in Australia rather than *vice versa*.

[21] ERA, Draft Decision on Proposed Revisions to the Access Arrangement for the Dampier to Bunbury Natural Gas Pipeline, March 2011, p.114

[22] Which references the guidelines which references the DBNGP final decision which references the DBNGP draft decision.

40. In any event, there are now at least three studies of the zero beta premium that exists in Australia CEG (2008),[23] NERA (2012) [24] and SFG (2014)[25]. These studies find a zero beta premium of between half and 100% of the market risk premium.

3.1.4 Peer review published versus not published studies

41. The ERA has argued that the Australian studies are not published in a quality journal and therefore the results cannot be relied on. This view fails to come to terms with the fact that the issue has been settled in the finance literature. This issue is not a matter of current academic study.

42. A researcher that performed such a study today in an attempt to publish it would be met with the question: what does this study add to the field that hasn't already been established starting in 1972 with Black Jensen Scholes? On the other hand, a study that robustly found a zero beta premium would be of considerable interest and would likely find publication in a top quality journal.

43. This is the reason that regulated businesses have been forced to commission the three studies that have been performed using Australian data. By requiring that an Australian study be published in a quality journal before the ERA will have regard to it the ERA is, perversely, creating a barrier to accepting the magnitude of the zero beta premium that is based on the fact that the existence of a zero beta premium is a settled issue in the finance literature.

44. Moreover, as discussed in relation to the FFM model, the ERA has performed its own study of the FFM (not published in a journal) and, on that basis, rejected having regard to the FFM despite the existence of a published article that reaches different conclusions.[26] I make two observations about this:

    - First, it is not clear why the ERA performed its own study of the FFM but not the Empirical CAPM – when it previously foreshadowed it would do the latter;
    - Second, it would be internally inconsistent to reject studies performed for submitters on the basis that these studies are not published in a peer reviewed journal while, the same time, preferring an unpublished ERA study of the FFM to a published study of the FFM in Australia.

[23] CEG, Estimation of, and correction for, biases inherent in the Sharpe CAPM formula, September 2008.

[24] NERA Economic Consulting 2012, The Black CAPM: A report for APA Group, Envestra, Multinet & SP AusNet is cited on page 117 of the ERA's Explanatory Statement for the Draft Rate of Return Guidelines.

[25] SFG, Cost of equity in the Black Capital Asset Pricing Model, May 2014.

[26] Brailsford, T., C. Gaunt, and M. O'Brien, 2012. "Size and book-to-market factors in Australia," Australian Journal of Management, 37, 261–281.

3.1.5 Credibility of results

45. The other reason that the ERA provides for rejecting the Empirical CAPM is that results from the NERA study that it reviewed found that the zero beta premium was indistinguishable from the market risk premium. The ERA appears to have rejected this result as not credible. [27]

> *Of most concern to the Authority is the inference from NERA's modelling that the zero beta portfolio has a return that is consistent with the MRP, leaving no room for variation in returns across stocks based on relative risk. The Authority considers that it is difficult to reconcile this result with either the theoretical underpinnings of portfolio theory, or actual observed outcomes in the market.*

46. In this statement the ERA is making one of two errors. It is either:

- incorrectly interpreting the NERA finding as suggesting that all firms have the same risk; or
- it is unreasonably starting from a prior assumption that beta must be an important determinant of risk and that, therefore, any study that finds it is not must be wrong.

47. In relation to the first dot point, this is not a correct description of what the NERA study found. The NERA finding is that a stock's (or a portfolio of stocks') risk is not determined by the measured equity beta. This is not the same thing as saying that all stocks have the same risk – just that equity beta is not a good measure of that risk.

48. Moreover, this is not an incredible finding. This is a finding that has been found in varying degrees in many studies. For example, Da, Guo and Jagannathan (2012)[28] provide evidence in support of a zero beta premium of 77% of the market risk premium. Campbell and Vuolteenaho (2004)[29] have estimated that the return on zero beta equity is above the market return. (It should be noted that neither of these studies set out to prove the existence of a zero beta premium – the estimates are by product of a different research agenda).

49. Rejecting an empirical finding because it does not accord with a prior belief is, as a general matter of principle, unsound. It is especially unreasonable when that empirical finding is consistent with other studies and your prior belief is not consistent with any empirical studies.

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[27] ERA , Explanatory Statement for the Draft Rate of Return Guidelines, Appendix 8, p. 66 paragraph 47.

[28] Da, Zhi, Re-Jin Guo and Ravi Jagannathan. 2012. CAPM for Estimating the Cost of Equity Capital: Interpreting the Empirical Evidence. Journal of Financial Economics. 103(1): 204-220

[29] Campbell, John Y. and Tuomo Vuolteenaho, 2004, "Bad beta, good beta," American Economic Review 94, pp. 1249-1275

3.2 The FFM

50. The basis of the ERA's rejection of the use of the FFM is twofold and is reflected in the below quote:[30]

> *680. The Authority notes that the Fama French three factor model (FFM) has consistently been put forward by regulated businesses as a means to estimate the return on equity. However, in its previous regulatory decisions, the Authority concluded that there is no strong theoretical basis to support the inclusion of the two additional risk factors to estimate the rate of return on equity, as occurs in the FFM. This is because the FFM is dependent on empirical justification – that is, the systematic observance of the FFM risk premia. In contrast, given that the FFM risk premia are not systematically observed in the Australian market, there is no reasonable basis for the FFM to be applied in Australia.*
>
> *681. The Authority's recent analysis of the FFM in the context of the Australian market for equity, for this Draft Decision, shows that observed empirical evidence is not consistent with the FFM (refer to Appendix 4 of this draft decision).*

51. In essence, the ERA gives no weight to the FFM on the grounds that: a) there is no strong theoretical basis to support the inclusion of the non-beta risk factors; and b) the ERA's analysis using Australian data is not consistent with the FFM.

52. On the first ground, I consider that it is unreasonable to prefer the Sharpe Lintner CAPM to the FFM on the grounds that the Sharpe Lintner CAPM is solidly based in theory and the others are based on empirical relationship. This approach is not scientific. Theory must be informed by fact and when a theory is falsified by the facts it must be adapted in a manner that is more consistent with the facts.

53. An analogy could be drawn with the natural sciences. The medieval Church had a theory of the operation of the solar system, based on humanity's assumed central role in the universe, which had the earth at the centre and the sun and other planets revolving around the earth. Copernicus, Galileo and Kepler, based purely on empirical observation and not theory, came to the conclusion that the sun was at the centre of the solar system and not the earth. At least initially, no theory existed to explain this empirical finding. Nonetheless, it was correct and was a critical input into the work of astronomers and navigators.

54. Two hundred years after Copernicus, a theory was developed that closely explained the empirical facts. This was the theory of gravity as described by Isaac Newton. However, the absence of a theory to explain the observed empirical facts did not provide a reasonable basis to reject those facts in the meantime. Assuming that the

[30] ERA, ATCO draft decision, p. 157.

earth was at the centre of the solar system was not reasonable in the face of the evidence – even without a unified theory to explain the evidence.

55. For the same reasons it would be an error to assume that beta is the sole determinant of risk (to assume that the Sharpe Lintner CAPM was true) when the available evidence suggests that risk is better explained by other factors. This is true even if these other factors have been identified empirically rather than theoretically. Having identified that these factors are important determinants of risk, this suggests a research agenda that tries to understand why this is the case – and to potentially refine the measures (as Einstein's theory of relativity is a refinement on Newtonian physics). However, the fact that this research has not yet resulted in a unified theory of risk consistent with the empirical data is not a reason to reject the empirical data.

56. The other reason relied on by the ERA is its own an empirical study that the ERA believes shows that the FFM does not explain Australian stock market returns. Let us accept for a moment that the ERA study is robust and that the FFM does not explain Australian stock market returns. I agree with the ERA that, if this is the case, then the FFM should not be used. However, by precisely the same logic, the ERA should test the Sharpe Lintner CAPM and, if the Sharpe Lintner CAPM does not explain Australian stock market returns, it should also be rejected in favour of the Empirical CAPM. The ERA could have used the same time periods to test the Sharpe Lintner CAPM and compare the results. However, it did not do so. Moreover, the ERA has not only not attempted such a study itself it has also rejected having regard such studies by third parties.

57. In my view, the ERA has acted in an internally inconsistent and unreasonable manner in relying on an empirical test of the FFM while simultaneously failing to apply a similar test to the Sharpe Lintner CAPM and rejecting the relevance of such tests performed by other parties.

58. Turning now to the robustness of the ERA study, I note that the study has serious flaws that, in my view, invalidated its findings. In particular, the study used only 5 years of data. This is simply insufficient data to do any useful analysis. The classic Black, Jensen and Scholes (1972) study of the Sharpe Lintner CAPM used data from 1931 to 1965. CEG (2008) using Australian data covered the period 1964 to 2007. Brailsford, Gaunt and O'Brien (2012)[31] used data from 1982 to 2006 in its test of the FFM. The ERA's study simply does not have sufficient data to be relied on.

59. The justification supplied by the ERA for using 5 years is as follows.

> *As a standard Australian regulatory control period is 5 years, estimates of parameters in the calculation of a rate of return are generally conducted*

[31] Brailsford, T., C. Gaunt, and M. O'Brien, 2012. "Size and book-to-market factors in Australia," Australian Journal of Management, 37, 261–281

> *every 5 years. As such, daily data of stock and market returns for the 5 year period from 1 July 2009 to 31 May 2014 are adopted.*[32]

60. This is not a sensible justification for using only five years of data. The length of the regulatory period has no bearing on the amount of data that should be used to test an asset pricing model.

61. Based on its study the ERA concluded that

> *The findings from this second stage regression indicate that the coefficients on HML are all statistically significant at 1 per cent level of significance. However, these coefficients are all negative which is not an expectation from the Fama French three factor model and findings from the recent Australian study by Brailsford; Gaunt & O'Brien (2012b).*

62. Here the ERA recognises that its results are different to those of Brailsford; Gaunt & O'Brien (2012). The later study, as already described, uses 24 years of data and is published in a peer review journal. Nonetheless, and despite the methodological shortcomings of the ERA study, the ERA relies on this study to conclude that: [33]

> *… observed empirical evidence is not consistent with the FFM (refer to Appendix 4 of this draft decision).*

63. For the reasons set out above I do not regard this as a reasonable conclusion.

[32] ERA, ATCO draft decision, p. 372.

[33] ERA, ATCO draft decision, p. 157.

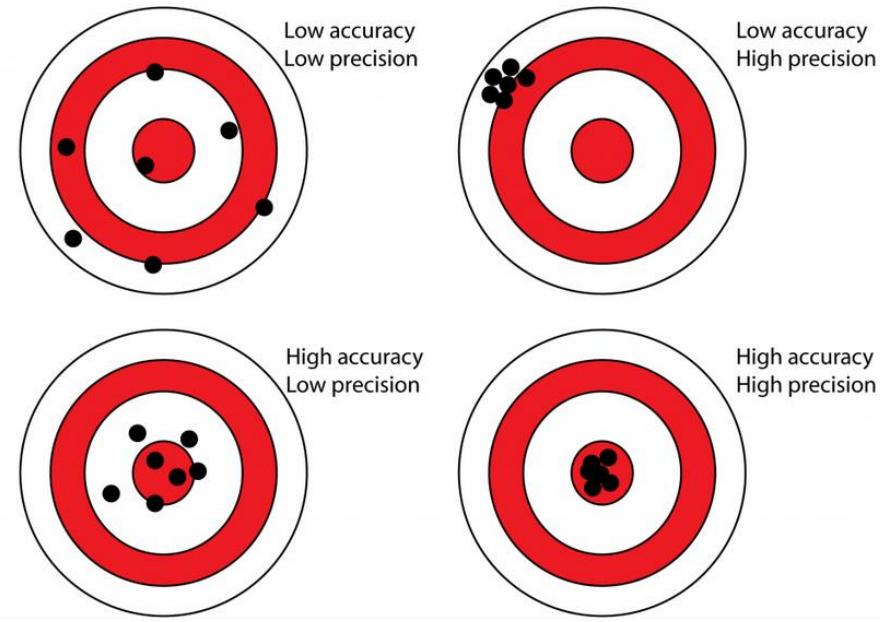
4 Accuracy vs precision

64. A theme running through the ERA's treatment of both the Empirical CAPM and the FFM is that the ERA appears to prefer precision over accuracy. In drawing a distinction between precision and accuracy I mean to distinguish between:

    - A precise method which is one where there is relatively little uncertainty around the answer that the method will give – but that answer may not be the right answer; and
    - An accurate method is one that is centred on the right answer but there may still be imprecision in the answer (i.e., uncertainty around the answer that the method will give).

65. This is illustrated in the below graphic which shows different combinations of precision and accuracy – where the middle of the 'bullseye' is the correct answer.

Figure 1: Precision vs Accuracy



66. I would characterise the ERA's sole reliance on the Sharpe Lintner CAPM as representing, in the ERA's judgement, the example in the top right hand corner of Figure 1 – precise but inaccurate. I would characterise its objection to the Empirical CAPM as an aversion to a scenario like that in the bottom left hand corner – more accurate but less precise.

67. The acceptance by the ERA that low beta bias exists, but the failure of the ERA to attempt to quantify the low beta bias using the Empirical CAPM in conjunction with the statement at paragraph 755 of the ATCO draft decision that:

> *The Authority is of the view that it is difficult to quantify the extent of any downward bias.*

is indicative of a preference for a simpler (theory based) estimate of the cost of equity that, despite being inaccurate, is thought to be precise. This approach can be thought to be precise because it avoids having to grapple with empirical facts – which can be messy and will lead to imprecision even as it improves accuracy.

68. I consider that this is not a sound basis for the regulator to proceed. A regulator should choose to give weight to different estimation techniques in a manner designed to minimise the expected error in the estimate. This does not necessarily mean giving 100% weight to the most accurate method (the method with the least bias). This is because if this method is imprecise an individual estimate may still have a large error term.

69. The weight that should be given to any individual estimate depends on a trade-off between the bias and the precision of the estimate. In statistical terms, the weights should be chosen to minimise the mean squared error (MSE) of the ultimate estimate. The mean squared error is the variance of the estimate (a measure of the precision of the estimate)[34] plus the square of the bias of the estimate.[35] Lally has recently set out similar views in relation to estimation of the cost of debt – where he recommends equal weight be given to two different methods for estimating the cost of debt.[36]

70. Lally argues that with two different estimators ($\widehat{T_1}$ and $\widehat{T_2}$ with mean squared errors of MSE_1 and MSE_2) the weight that should be given to $\widehat{T_1}$ (i.e., the weight that minimises the MSE) is "w" and w should be set to minimise MSE. He argues this is achieved where w is set equal to:

$$w = \frac{MSE_2 - Cov(\hat{T}_1, \hat{T}_2)}{MSE_1 + MSE_2 - 2Cov(\hat{T}_1, \hat{T}_2)}$$

71. Lally concludes on this basis that, if the estimators have the same MSE, which he assumes that they do, then equal weight should be given to them. It is on this basis that Lally recommends equal weight be given to the Bloomberg and RBA fair value estimates of the cost of debt.

[34] The expected value of the squared difference between the mean of all estimates using that method and each individual estimate.

[35] The squared difference between the true value being estimated and the mean of the estimates using the method in question.

[36] Lally, Implementation Issues for the Cost of Debt, Capital Financial Consultants Ltd 20 November 2014.

72. However, the same logic can be applied to the weights that should be given to the Sharpe Lintner CAPM, the Empirical CAPM and the FFM. The estimates with the smallest MSE should be given the most weight. However, all estimates should be given some weight unless one of them has a zero MSE. That is, it is an extreme approach to give one estimator 100% weight – even if it is regarded as having the lowest MSE.

73. Moreover, in my view the *a priori* best estimate would be that the Sharpe Lintner CAPM has the highest MSE – and therefore should be given the least weight. This reflects the fact that ERA has acknowledged that the Sharpe Lintner CAPM is biased for low beta stocks. By contrast, the potential for bias in the Empirical CAPM is smaller precisely because it is calibrated to remove the bias present in the Sharpe Lintner CAPM. On this basis the Empirical CAPM will have a lower MSE than the Sharpe Lintner CAPM.[37]

74. Similarly, the FFM is also an empirical model and, as such, the potential for bias is smaller. However, unlike the Empirical CAPM, it has regard to more evidence when estimating returns (the size and book to market factors as well as beta). Whether the MSE of the FFM will be higher or lower than the Empirical CAPM depends on whether the additional accuracy associated with introducing the two new factors is offset by any additional imprecision associated with the estimation of those factors.

[37] Because it has the same other parameters (beta and MRP) but has an unbiased estimate of the return on zero beta stock. Thus, an identical variance is associated with estimation of beta and MRP but the estimation of the zero beta return is unbiased.

Appendix A Grundy survey of literature

75. The below passage is taken from Grundy, The Calculation of the Cost of Capital, A Report for Envestra, September 2010.

> *I know of no published study that has empirically tested the Sharpe Lintner CAPM and failed to reject the Sharpe Lintner CAPM. Table 1 sets out a number of studies cited by the AER in rejecting the use of the Fama-French 3 factor model (FFM) to determine required returns on stocks.*[38] *The FFM links the expected return on stock to three factors: the beta of the stock, the equity value of stock (the size of the stock) and the book-to-market ratio of the stock.*
>
> *Some of the papers cited by the AER are pure theory papers, while others are empirical studies of the relation between risk and return. Part A of Table 1 sets out the implications of the pure theory studies for the question of whether required returns are better-described by the Sharpe Lintner CAPM or the Black CAPM.*

TABLE 1
Part A: Pure theory papers cited by the AER in rejecting the FFM

Paper cited by AER	
Ferson, Sarkissian and Simin (1999)	Theoretical result: The genesis for the theoretical examination in this paper is that the FFM provides a better empirical fit to the data than is provided by the Sharpe Lintner CAPM. Suppose that: i) Average returns are related to a stock's beta, size and book-to-market ratio (i.e., to the 3 factors of the FFM), and ii) Average returns are related to a stock's sensitivity to the market and to proxies for a 'size factor' and 'book-to-market factor' and hence can be given a risk-reward interpretation. Despite i) and ii) being true, it may be that the higher average returns empirically observed on small stocks and on stocks with high book-to-market ratios are the result of a behavioural bias of investors rather than a reward for risk. Empirical implication for the Black CAPM: The Black CAPM will be a better predictor of stock returns than is the Sharpe Lintner CAPM provided low beta stocks tend to be

[38] The FFM is discussed in

Fama, Eugene F., and Kenneth R. French, 2004, "The capital asset pricing model: Theory and evidence," *Journal of Economic Perspectives* 18(3), pp. 25-46.

	smaller stocks and/or tend to have higher book-to market ratios.
Lo and MacKinlay (1990)	Theoretical result: If properties of the data are known to those developing theories and if the resultant model is then tested on data that consciously or otherwise provided the genesis for the model, it can appear to be a better model than it subsequently proves to be. Empirical implication: The relative ranking of the FFM, Sharpe Lintner CAPM and Black CAPM when explaining past returns may not be their relative ranking in the future. There is *no* implication that the Sharpe Lintner CAPM will provide a better model of required returns than the FFM or the Black CAPM.
Roll (1977)	Theoretical result: i) For any efficient portfolio $Re = R0^E + \beta^E(R^E - R0^E)$, where $R0^E$ is the average return on any stock that has zero beta with respect to the return on that efficient portfolio, β^E is the beta of the stock measured with respect to the return on that efficient portfolio, and R^E is the average return on that efficient portfolio. A portfolio is an efficient portfolio if it is the portfolio with the minimum volatility within the set of all portfolios with a given level of expected return. ii) According to the Sharpe Lintner CAPM, the true market portfolio containing all bonds, stock and real estate is an efficient portfolio. iii) The stock market alone may not be an efficient portfolio. Empirical implication: Since the expected return on the stock market exceeds the expected return on the true market portfolio of all bonds, stock and real estate, then it is the case that even if the stock market is an efficient portfolio, the return on equity will be given by the Black CAPM and not by the Sharpe Lintner CAPM.
Roll and Ross (1994)	Theoretical result: The genesis for the theoretical examination is according to the paper's abstract that "empirical research has found little relation between sample mean returns and estimated betas." Thus the paper is motivated by the empirical observation that the relation between returns and betas is flatter than predicted by the Sharpe Lintner CAPM). The paper shows theoretically that such a near flat relation can arise even if the stock market is very close to an efficient portfolio. Empirical implication: The return on equity may be well-described by the Black CAPM even though it is not well-described by the Sharpe Lintner CAPM.

None of the four theoretical papers cited by the AER in rejecting the FFM provides any basis for a claim that the Sharpe Lintner CAPM theoretically dominates the FFM. Consider Ferson, Sarkissian and Simin (1999). That paper's theoretical explanation for the empirical superiority of the FFM over the Sharpe Lintner CAPM as potentially due to a behavioural bias by investors rather than a reward for risk does not challenge the empirical observation that gave rise to the paper—namely that the FFM provides a better empirical description of stock returns than the Sharpe Lintner CAPM does. Note also that all four theoretical papers cited by the AER in rejecting the FFM are consistent with the Black CAPM providing a better descriptor of average stock return than the Sharpe Lintner CAPM; in fact, the Roll and Ross (1994) analysis is motivated by exactly this empirical observation.

Part B of Table 1 sets out the results reported in those studies cited by the AER in rejecting the FFM that undertake an empirical examination of the link between risk and return. Part B of Table 1 also sets out the results in two classic tests of the Sharpe Lintner CAPM: Fama and Macbeth (1973) and Black, Jensen and Scholes (1972). Column 1 contains the author names and the year of publication of the study. Column 2 contains the sample period examined. Column 3 sets out the likelihood that the Sharpe Lintner CAPM is true given the data examined by the authors. Where it is possible to determine the ratio $\frac{Rm - R0}{Rm - Rf}$ *from the results reported in the paper, column 4 reports the estimated value of this ratio. The notation n.a. denotes that this ratio could not be calculated from the results reported in the paper.*

TABLE 1
Part B: Empirical papers cited by the AER in rejecting the FFM
plus 2 classic tests of the Sharpe Lintner CAPM

paper	Sample Period		$\frac{Rm - R0}{Rm - Rf}$
Empirical papers cited by the AER			
Schrimpf, Schröder and Stehle (2007)	1969 - 2002	Estimate of $Rm - R0 = 0.2\%$ per month. Note than an annual MRP of 6.5% implies a monthly MRP of 0.54% per month.	n.a.
Ang and Chen (2007)	1926 - 1963:06	Cannot reject the Sharpe Lintner CAPM	n.a.
	1963:07 -	Likelihood the Sharpe Lintner CAPM true is <	n.a.

	2001	1%	
Gruaer and Janmaat (2010)	1963 - 2005	For 7 of the 14 methods for grouping stocks to form portfolios that are examined in the paper, the likelihood of the Sharpe Lintner CAPM being true is < 5%	n.a.
Gregory and Michou (2009)	1975 - 2005	Examines 35 industries. For only 3 industries would one reject the Sharpe Lintner CAPM at the 5% level. For the *Gas, Water and Multi-utility Industry* returns are statistically significantly higher at the 5% level than predicted by the Sharpe Lintner CAPM	n.a.
Black (1993)	1926 - 1965	likelihood Sharpe Lintner CAPM true < 1%	n.a.
Schwert (2003)	1926 - 2001	likelihood Sharpe Lintner CAPM true < 0.0001%	n.a.
Morana (2009)	1965 - 2001	likelihood Sharpe Lintner CAPM true < 1%	n.a.
Daniel, Titman and Wei (2001)	1975 - 1997	likelihood Sharpe Lintner CAPM true < 0.34%	n.a.
Da, Guo and Jagannathan (2009)	1932 - 2007	likelihood Sharpe Lintner CAPM true < 0.002%	0.232
Kothari, Shanken and Sloan (1995)	1927 - 1990	likelihood Sharpe Lintner CAPM true < 0.058%	0.415
Classic tests of the Sharpe Lintner CAPM			
Fama and Macbeth (1973)	1935 - 1968	likelihood Sharpe Lintner CAPM true < 0.55%	0.639
Black, Jensen and Scholes (1972)	1931 - 1965	likelihood Sharpe Lintner CAPM true < 0.0001%	0.761
		Average = 0.511	

Schrimpf, Schröder and Stehle (2007) do not test whether the Sharpe Lintner CAPM fits the data. Rather they conclude only that FFM does not fit the data better than the Sharpe Lintner CAPM does. For the first half of the sample period examined by Ang and Chen (2007) the authors do not reject the Sharpe Lintner CAPM. The authors do reject the Sharpe Lintner CAPM using data after 1963.

Although Gregory and Michou (2009) do not reject the Sharpe Lintner CAPM for most industries, the nature of their test bears discussion. Gregory and Michou regress the monthly return on an industry portfolio on the monthly return on the market. Most industries have betas near one and both the Sharpe Lintner CAPM and the Black CAPM make the same prediction for stock with a beta of one; the expected return on a beta one stock equals the expected return on the market. Gregory and Michou do not reject this prediction. Interestingly, for the portfolio whose beta is furthest from one, namely the Gas, Water and Multi-utility Industry, stock

returns are significantly higher (at the 5% level) than predicted by the Sharpe Lintner CAPM. This is consistent with the true relation between expected returns and betas being flatter than the relation predicted by the Sharpe Lintner CAPM.

Every other study listed in Table 1B rejects the Sharpe Lintner CAPM and does so because the estimated return on a zero beta stock, R0, exceeds the risk-free rate, Rf. Equivalently, in every case the estimated difference in the return on the market and the return on zero beta stock is significantly less than Rm – Rf. Thus every other study documents that the thick line of Figure 1 is flatter than the thin line of Figure 1; i.e., that the empirical relation between the cost of equity and beta is flatter than is predicted by the Sharpe Lintner CAPM.

Where the paper's reported results make it possible to calculate the average values of (Rm – R0) and (Rm – Rf) over the sample period, the ratio of the two average differences is reported in column 4. Averaging over the four papers where this possible, the difference between the return on the market and the return on zero beta stock is only 0.511 of the difference predicted by the Sharpe Lintner CAPM.

The full citations for the set of papers in Table 1 are given below in the order the papers are listed in the table:

Ferson, Sarkissian and Simin, 1999, "The alpha factor asset pricing model: A parable," Journal of Financial Markets 2, pp. 49-68

Lo, Andrew W. and A. Craig MacKinlay, 1990, "Data-snooping biases in tests of financial asset pricing models," Review of Financial Studies 3(3), pp. 431-467.

Roll, Richard, 1977, "A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory," Journal of Financial Economics 4(2), pp. 129–176.

Roll, Richard and Stephen A. Ross, 1994, "On the cross-sectional relation between expected returns and betas," Journal of Finance 49(1), pp. 101-121.

Schrimpf, Andreas, Michael Schröder and Richard Stehle, 2007, "Cross-sectional tests of conditional asset pricing models: Evidence from the German stock market," European Financial Management 13(5), pp. 880–907.

Ang, Andrew and Joseph Chen, 2007, "CAPM over the long run: 1926–2001," Journal of Empirical Finance 14, pp. 1–40.

Grauer, Robert R. and Johannus A. Janmaat, 2010, "Cross-sectional tests of the CAPM and Fama–French three-factor model," Journal of Banking & Finance 34, pp. 457–470.

Gregory, Alan and Maria Michou, 2009, "Industry cost of equity capital: UK evidence," Journal of Business Finance & Accounting 36(5) & (6), pp. 679–704.

Black, Fischer, 1993, "Beta and return," Journal of Portfolio Management, 1993, 20(1), pp. 8–18.

Schwert,G. William, 2003, "Anomalies and market efficiency," in Handbook of the Economics of Finance, editors G. Constantinides, M. Harris and R. Stulz, Elsevier Science, ch. 15, pp. 937–972.

Morana, Claudio, 2009, "Realized betas and the cross-section of expected returns," Applied Financial Economics, 19, pp. 1371-138.

Daniel, Kent, Sheridan Titman and K.C. John Wei, 2001, "Explaining the cross-section of stock returns in Japan: factors or characteristics', Journal of Finance, 56(2), pp. 743–767

Da, Zhi, Re-Jin Guo and Ravi Jagannathan, 2009, "CAPM: Interpreting the evidence," NBER working paper 14889.

Kothari, S., Jay Shanken and Richard G. Sloan, 1995, "Another look at the cross-section of expected returns," Journal of Finance, 50(1), pp. 185–224;

Fama., Eugene F. and James D. Macbeth, 1973, "Risk, return, and equilibrium: Empirical tests," Journal of Political Economy, 81(3), pp. 607-636.

Black, Fischer, Michael C. Jensen and Myron S. Scholes, 1972 "The capital asset pricing model: Some empirical tests," in Studies in the Theory of capital Markets, Michael C. Jensen editor, (Praeger Publishers Inc.).