CRITIQUE OF AVAILABLE ESTIMATES OF THE CREDIT SPREAD ON CORPORATE BONDS

A Report for the ENA

Prepared by NERA

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EXECUTIVE SUMMARY

Regulators use the interest rate a firm would have to promise on its bonds if it were to raise capital through the issuance of 10 year debt in estimating the efficient cost of debt. In practice, there are very few long dated, low rated bonds outstanding and one can rarely directly observe yields on 10 year debt.

Many regulators have relied on CBASpectrum which provides estimated yields on 10 year debt. The CBASpectrum estimation procedure does not determine the best fit to the available data. The CBASpectrum estimation procedure is such that CBASpectrum estimated yields are expected to be, and in practice are, on average, less than actual yields for long dated, low rated bonds. Between 30 June 2003 and 10 May 2005, actual yields on Australian bonds with more than 6 years to maturity and ratings of A or below averaged 17.1 basis points higher than CBASpectrum estimated yields on such bonds. For bonds with more than 8 years to maturity and ratings of A or below, the difference has averaged 22.2 basis points.

On this basis, we consider that the minimum reasonable adjustment to CBASpectrum estimates by regulators seeking to estimate the cost of debt on 10 year low rated debt is 22.2 basis points.[1] Using only data from CBASpectrum, our best estimate of the appropriate adjustment to CBASpectrum estimates of yields on 10 year debt rated A or below is to add 25.6 basis points.

The regulator might consider relying on a different data source such as Bloomberg. Bloomberg's estimation procedure will not induce an expected difference between actual and estimated yields. Over the relevant period Bloomberg estimated fair yields that were 25.8 basis points higher than CBASpectrum for ten year BBB+ bonds. This is consistent with our independent assessment of a 25.6 basis point underestimate.

Finally, regulators may wish to engage in a consultation process in order to develop a 'tailor made' transparent estimation technique to derive unbiased estimates of the cost of debt on long dated low rated corporate bonds.

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[1] It should be understood that we have not, and do not in this report, express an opinion regarding the appropriateness of other uses for the CBASpectrum product. We note that the vast majority of corporate bonds on issue have relatively short terms to maturity and are relatively highly rated. We do not expect CBASpectrum's estimates of the fair yield on these bonds to have any material bias. Thus, the issues we have identified at likely to be peculiar to the use of CBASpectrum by Australian regulators and are unlikely to be relevant for other uses of that product.

1 INTRODUCTION AND BACKGROUND

1.1 Introduction

This report provides an opinion on the potential for CBASpectrum 'fair value' credit spreads to understate credit spreads observed in the marketplace. It also compares, in the context of setting the cost of debt for regulated businesses, the relative merits of CBASpectrum versus Bloomberg estimates of credit spreads for long dated and low rated bonds. It also provides suggestions about alternatives to the use of CBASpectrum by regulators.

In preparing this report we have had regard to documentation detailing CBASpectrum's estimation procedures entitled '***Spectrum Re-built Analytics***' supplied by CBASpectrum to Professor Bruce Grundy in June 2004. We have also had regard to documentation describing Bloomberg's estimation technique entitled '***Market Solutions for Pricing and Analytics, the Curve's the Thing***' and telephone and email correspondence with Bloomberg staff.

1.2 Background

CBASpectrum produces estimates of the average (or 'fair value') yield on corporate bonds of a particular credit rating and of a particular maturity. These estimates of 'fair value' yield have been used by a number of Australian regulators in setting the 'efficient' cost of debt for regulated businesses.

In mid 2004 NERA observed that the CBASpectrum database contained only 3 corporate bonds rated A or below with longer than 6 years to maturity and for those three bonds the average difference between CBASpectrum 'fair value' and actual yields was consistently around +18 basis points over the preceding 12 months. Following this observation we endeavoured to better understand CBASpectrum's estimation technique for 'fair value' yields on corporate bonds. We asked CBASpectrum if they could provide documentation describing the estimation technique used. In response to this request we received a document entitled "Spectrum Re-built Analytics".

Upon examination of this documentation we formed the opinion that CBASpectrum's method of determining 'fair value' yields leads CBASpectrum yields to understate actual yields on long dated/low rated bonds. The reasons for this were set out in TransGrid's submission to the ACCC on the weighted average cost of capital and in a report by Professor Grundy – both of which were made confidential in anticipation that the issues involved would be resolved with the assistance of CBASpectrum. We further formed the opinion that this meant that Australian regulators' current use of CBASpectrum to set the regulatory cost of debt is inappropriate.

It should be understood that we have not, and do not in this report, express an opinion regarding the appropriateness of other uses for the CBASpectrum product. We note that the vast majority of corporate bonds on issue have relatively short terms to maturity and are relatively highly rated. We do not expect CBASpectrum's estimates of the fair yield on these bonds to have any material bias. Thus, the issues we have identified at likely to be peculiar to the use of CBASpectrum by Australian regulators and are unlikely to be relevant for other uses of that product.

This report covers much of the same ground as previous confidential submissions for TransGrid. It is made with the aim of assisting the ENA to engage with Australian regulators and CBASpectrum on this issue.

All of the Bloomberg and CBASpectrum data in this report was sourced from an ENA member. NERA is responsible for the correct manipulation and economic interpretation of that data.

2 WHY ACTUAL SPREADS ARE CONSISTENTLY HIGHER THAN CBASPECTRUM FITTED SPREADS

The first equation in section 2 of the '***Spectrum Re-built Analytics***' document states that the CBASpectrum estimation technique fits a set of curves used to predict 'fair value' yields on corporate bonds such that the fitted curves (of a predetermined functional form) minimise the sum of squared deviations:

a. between CBASpectrum estimates and actual observations (ie, the standard statistical technique); plus the sum of squared deviations

b. between CBASpectrum estimates at 10 years to maturity of 'fair value' yields for each credit rating and the CBASpectrum estimate of the 'fair value' yield for the next highest credit rating at 10 years to maturity; plus the sum of squared deviations

c. between the CBASpectrum estimate for CGS (Commonwealth Government Securities) and BBB- bonds at the 1 year maturity.

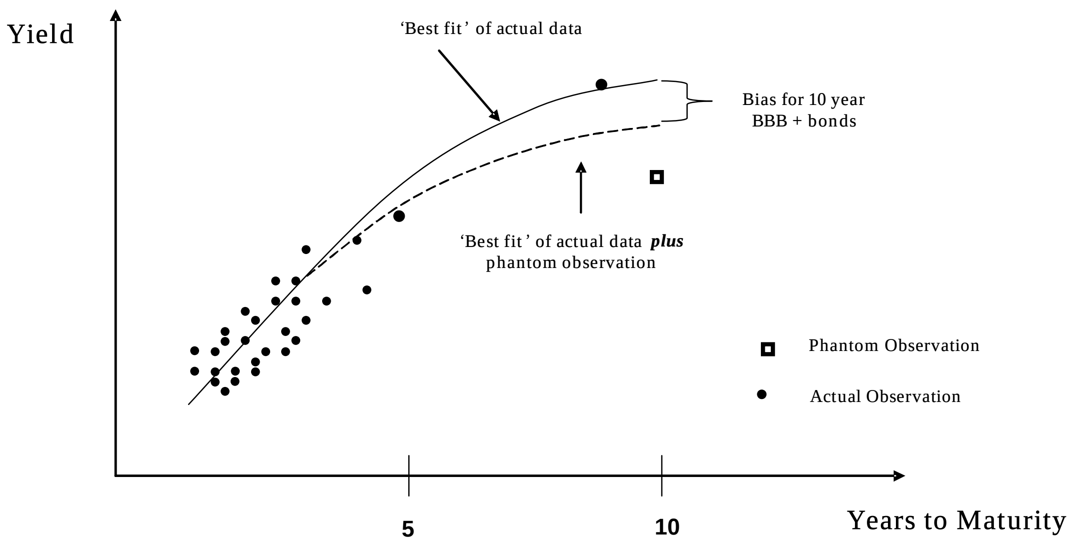
The procedure described in a) above represents the standard statistical technique where curves are fitted in a manner that minimises the deviation between the predictions of the model and the ***actual data*** available. Combining b) with a) represents a departure from this standard technique and effectively introduces, for each credit rating, one 'phantom' observation of a 10 year corporate bond with a credit spread that is the same as that predicted for the next highest rated bond of 10 years. For example, it introduces a phantom observation of a BBB rated 10 year corporate bond that has a credit spread that is equal to CBASpectrum's estimated credit spread for BBB+ 10 year bonds. Similarly, it introduces a phantom observation of a BBB+ 10 year bond that has a credit spread equal to CBASectrum's estimated credit spread for A- 10 year bonds (and so on for higher credit ratings).[2]

This aspect of the estimation technique is particularly likely to cause estimates of 'fair yields' for long dated corporate bonds to be below these bonds' actual yields because of the small number of observations of long dated corporate bonds – and especially low rated and long dated corporate bonds. In mid 2004 there were only be three corporate bonds rated A or below with longer than 6 years to maturity in the CBASpectrum database. In the absence of actual observations of long dated and low rated corporate bonds, CBASpectrum's estimation technique will place significantly more weight on the 'phantom' observations when fitting the long maturity end of the curve.

[2] Moreover, combining, point c) with a) and b) effectively introduces a further 'phantom' observation of a 1 year corporate BBB- bond with a zero credit spread. In other words, the methodology estimates the relevant equation parameters in the same way that the standard procedure would if there really was an observation of a one year BBB- bond with a zero credit spread – despite there being no such actual observation.

The impact of these phantom observations on the estimation technique can be illustrated graphically.

'Fair Yield' on BBB +



The above graph depicts two alternative curves which estimate the 'fair yield' on BBB+ corporate bonds. The 'solid' curve represents the curve that best fits the actual observations of yields on BBB+ bonds (represented by the round dots) – ie, the curve that minimises the sum of squared deviations of actual observations from the fitted curve. The dashed curve represents the curve that best fits the actual observations ***plus*** the phantom observation as described in part b) of the first paragraph of this section (marked by a 'square dot'). This phantom 10 year BBB+ observation is biased downward because it is assigned the same yield as CBASpectrum estimates for a 10 year A- bond (which is itself biased downwards).

As drawn, both curves provide a near identical fit to the actual observations of BBB+ bonds with 5 year and below maturity dates. However, the two curves begin to materially depart for longer dated maturities – with their maximum departure occurring at 10 years. The reason for this divergence is that the dotted curve is 'pulled' towards the phantom observation which, by definition, has a low yield (equal to the estimated yield on the next highest rated bond). Moreover, because there is only one actual observation of BBB+ bonds with greater than 5 years to maturity there is only weak 'pull back' by the actual observations. That is, because there is only one actual observation of a long dated BBB+ bond, the dotted curve effectively gives the 'phantom' observation equal weight with the actual observation - and fits a line "halfway" between them. If there were more actual observations of long dated BBB+ bonds, then the phantom observation would have less impact.

The above discussion illustrates with reference to a single credit rating (BBB+) the reason why CBASpectrum's methodology causes fitted yields on long dated bonds to be below their actual yields. In reality, CBASpectrum estimates yield curves for all credit ratings, including CGS, simultaneously. There are a number of reasons why CBASpectrum may wish to estimate all yield curves simultaneously, including a desire to impose a restriction that yield curves for different credit ratings do not cross.[3] However, the simultaneous estimation of 'fair yields' for different credit ratings does not alter the fact that CBASpectrum's methodology causes fitted yields on long dated bonds to be below their actual yields.

In Appendix A to this report Professor Bruce Grundy provides a simulation showing that, whether the term structure of true spreads is flat, upward-sloping or downward-sloping, when we consider a distribution of observations on each rating/maturity class like the empirical distribution across ratings/maturity classes in the CBASpectrum database, the following results hold:

i. CBASpectrum estimated spreads are, on average, less than actual spreads.

ii. For a given rating, the excess of actual spreads over CBASpectrum estimated spreads is larger for longer maturity bonds than for shorter maturity bonds.

iii. For a given maturity, the excess of actual spreads over CBASpectrum estimated spreads is larger for lower quality bonds than for higher quality bonds.

iv. The excess of actual spreads over CBASpectrum estimated spreads is largest for long dated, low rated bonds.

Professor Kevin Davis has also reviewed CBASpectrum's documentation of its model and has agreed that there is a substantive argument that CBASpectrum's estimation technique creates a material downward bias in estimates of "fair spread" for long dated and low rated bonds. His report is included as Appendix B.

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3 For example, by estimating the curves simultaneously CBASpectrum can impose a restriction that the yield on BBB+ bonds is never lower than the yield on A- bonds of a similar maturity and that A- bonds never have yields less than the yields on A bonds of similar maturity and so on.

3 HISTORICAL DIFFERENCES BETWEEN ACTUAL SPREADS AND CBASPECTRUM FITTED SPREADS

The table below describes the historical difference between CBASpectrum estimates of 'fair yields' and actual yields for the three corporate bonds that, in mid 2004, were in CBASpectrum's database and were rated A or below with longer than 6 years to maturity.

Table 1
Prior Historical Analysis

Average of the Historical Excess of Actual Spreads over CBASpectrum Fitted Spreads on long-term bonds over the year ended 30 June 2004				
	Snowy Hydro 25/02/2013	Stockland 15/5/2013	Westfield 15/07/2010	Equal weighted portfolio
	BBB+	A-	A	
Last 12 Months (30 June 2003 to 30 June 2004)				
Average (bp)	32.16	15.56	6.48	18.71
SD (bp)	4.73	4.27	3.25	3.63
No. of positives	236	231	204	236
% of obs positive	100	100	94.01	100
No. of obs	236	231	217	236
Last 6 Months (1 January 2003 to 30 June 2004)				
Average (bp)	31.53	15.14	8.13	18.28
SD (bp)	3.94	3.74	2.50	2.55
No. of positives	116	115	115	116
% of obs positive	100	100	99.14	100
No. of obs	116	115	116	116
Last 1 Month (1 June 2004 to 30 June 2004)				
Average (bp)	28.62	16.52	10.00	18.38
SD (bp)	2.22	1.17	2.86	1.18
No. of positives	21	21	21	21
% of obs positive	100	100	100	100
No. of obs	21	21	21	21
Last 10 Trading Days (17 June 2004 to 30 June 2004)				
Average (bp)	27.20	16.00	11.50	18.23
SD (bp)	2.20	1.33	0.71	0.94
No. of positives	10	10	10	10
% of obs positive	100	100	100	100
No. of obs	10	10	10	10

This table has previously been supplied to the ACCC. The passage of time since 30 June 2004 allows us to examine a longer sample period and the results have been extended through to 10 May 2005. The average of the observed excess of actual yields over CBASpectrum fitted yields on these three bonds between June 30 2003 and 10 May 2005 has been 17.99 basis points. This small downward movement is likely to be, at least in part, a result of the reduction in the average time to maturity with the passage of time.

The reduction in the difference between actual and CBASpectrum fitted yields as a bond's the time to maturity decreases is further illustrated by noting that since 15 July 2004 the Westfield bond has had less than 6 years to maturity. When we remove the Westfield bond from this sample after 15 July 2004 and then recalculate the average difference between actual and fitted spreads between June 30 2003 and 10 May 2005 for this set of bonds (bonds rated A or below with greater than 6 years to maturity throughout the observation period) we obtain 18.81 basis points.

The passage of time has also increased the size of the sample of bonds rated A or below with longer than 6 years to maturity. There have been four new bonds issued that meet this criteria.[4] Adding these four bonds during the period over which they had at least 6 years to maturity to the existing sample of A or below rated bonds with at least 6 years to maturity and recalculating the average excess of actual yields over CBASpectrum fair value yields for the period from 30 June2003 to 10 May 2005, we estimate that the excess was 17.1 basis points. However, this estimate of average underestimate is unsatisfactory on two counts. First, it is based on bonds whose average time to maturity is 7.7 years. Second, the average credit rating of these bonds is between A- and A.

We understand that the object of interest for regulators is the size of the difference between actual yields and CBASpectrum fair value yields on bonds ***with 10 years to maturity***. We also understand that regulators are generally interested in the cost of debt on BBB+ bonds as this is the benchmark credit rating of a utility with benchmark gearing of 60%. Looking at the seven individual bonds in our sample, we see that the difference is much greater for bonds with nearly 10 years to maturity than it is for bonds with a only a little over 6 years to maturity.

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4 CFS Gandel Retail issued an A rated bond on 22 December 2004. The bond matures on 22 December 2014 maturity and CBASpectrum began reporting data on this bond on 5 January 2005. CFS Gandel Retail issued a second A rated bond on 12 November 2003. This bond matures on 12 November 2010 maturity and CBASpectrum began reporting data on this bond on 6 July 2004. By 12 November 2004 this bond had less than 6 years to maturity. SPI Electricity issued an A- bond on 3 November 2004. This bond matures on 3 November 2011 maturity and CBASpectrum began reporting data on this bond on 8 November 2004. Tabcorp issued a BBB+ bond on 13 October 2004. This bond matures on 13 October 2011 maturity and CBASpectrum began reporting data on this bond on 13 October 2004.

Table 2
Average Differences between Actual Yields and CBASpectrum Fitted Yields over the period 30 June 2003 through 10 May 2005 by Bond

Bond	CFS Gandel 2010	Westfield	SPI Electricity	Tabcorp	Snowy	Stockland	CFS Gandel 2014
Average years to maturity[5]	6.2	6.5	6.8	6.7	8.7	8.9	9.8
Credit Rating	A	A*	A-	BBB+	BBB+	A-	A
Average excess to CBASpectrum b.p.	12.41	6.77	8.91	4.37	28.43	14.75	23.42

*The Westfield bonds were downgraded to A- on 7 February 2005

The average excess to CBASpectrum is the average difference between the actual yield and the CBASpectrum fair value yield where that average was calculated over each date post 30 June 2003 on which the bond had at least 6 years to maturity. For the CFS Gandel Retail bond maturing in 2010, the average time to maturity when data on the bond was reported on CBASpectrum and the bond had at least 6 years to maturity was only 6.2 years —much less than 10 years. The average excess to CBASpectrum for this A rated bond was 12.41 basis points. Contrast this to the Snowy BBB+ bonds. This bond had on average 8.7 years to maturity during the post 30 June 2004 period and an average difference of 28.43 basis points.

The Table shows a very clear trend for the difference between CBASpectrum estimates of fair yields and actual yields to increase the longer the time to maturity. For the three bonds with an average maturity over the sample period between of eight and ten years, CBASpectrum underestimated these yields by an average of 22.2 basis points. For the four bonds with between six and eight years average maturity CBASpectrum's average underestimate was 8.1 basis points.

It is also worth noting that on all seven long dated and low rated bonds in the sample, CBASpectrum underestimated the average yield on each of these bonds. This empirical evidence strongly supports the theoretical presumption that CBASpectrum's technique for estimating yields will result in an underestimate for long-dated and low-rated bonds.

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5 Average years to maturity is calculated as the average of the bond's time to maturity with the average calculated over each date between:

- 30 June 2003 or the date on which CBASpectrum first began to report data on the bonds (whichever comes later); and
- 10 May 2005 or the date at which they bond first has less than 6 years to maturity (whichever comes earlier).

3.1 The Magnitude of the Underestimate on 10 year Bonds

We understand that an important question for regulators is: What is the fair yield on ten year bonds? In particular, if regulators were to rely on CBASpectrum analysis and data what adjustment would be appropriate to CBASpectrum's estimate of 'fair yield' on 10 year corporate bonds rated A or below?

On the basis of the previously outlined data, we consider that the minimum reasonable adjustment is 22.2 basis points. This represents the average underestimate of actual yields on all bonds in CBASpectrum's database rated A or below with between 8 and 10 years to maturity. We consider that this is an appropriate minimum adjustment on the grounds that the average time to maturity on these bonds is less than ten years (9.13 years) and we observe that the size of the underestimate tends to increase with maturity. The average rating for these three bonds is A-. However, within the sample of bonds rated A or below there does not appear to be any strong relationship between the size of the excess to CBASpectrum and the credit rating. As such, we consider that it is reasonable to apply the same adjustment to bonds rated A or below.

There is limited data for us to be definitive on the 'most likely' as opposed to 'minimum' adjustment required. However, three potential approaches all yield similar outcomes. In particular:

- straight line regression[6] of 'average excess' to 'average years to maturity' data from table 2 predicts an underestimate at 10 years to be 25.6 basis points;

- the coefficient of 'average years to maturity' in that regression is 4.9 basis points, ie, the excess tends to increase by 4.9 basis points per year. Adjusting for the fact that our minimum adjustment was based on an average age to maturity of 9.13 years by adding 4.9∗(10–9.13) basis points gives 26.4 basis points; and

- for the three bonds with between eight and ten years to maturity the average excess per year to maturity is 2.44 basis points per year. Adding 2.44∗(10–9.13) basis points gives 24.3 basis points.

We believe that the first of these estimates has the most rigour; however, it is comforting that alternative approaches provide similar estimates. On the above basis, our best estimate of the appropriate adjustment to CBASpectrum estimates of fair yield on A to BBB+ bonds with 10 years to maturity is to add 25.6 basis points.

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[6] The estimated relationship is Excess to CBASpectrum = -23.3bp + 4.898*(average years to maturity). The coefficient of 'average years to maturity' is statistically significant at the 95% confidence interval. Naturally, this relationship can only reasonably be relied on within the approximate range of the data used to derive it. That is, we do not claim that the relationship would be relevant for bonds with, say, 3 or 15 years to maturity.

4 IS BLOOMBERG FREE OF THE IDENTIFIED UNDERESTIMATE?

Bloomberg also provides estimates of the 'fair'/'representative' yields on corporate debt of a particular maturity and credit rating. In doing so, Bloomberg uses a different estimation technique. In particular, Bloomberg's technique solely minimises the sum of squared deviations between actual observations and estimates of fair/representative yields. It does not impose any additional 'penalties' on the estimation of 'fair'/'representative' yields, ie, there are no 'phantom' observations which the estimation technique attempts to 'fit'.

For this reason, Bloomberg's estimation technique does not introduce the tendency for estimated yields to understate actual yields that the CBASpectrum's estimation technique exhibits. That is, the technique fits estimated 'fair'/'representative' yields to minimise the sum of squared deviations from actual observations only.

This fact tends to be confirmed when we examine the excess of the yields on the seven bonds examined in Table 2 relative to Bloomberg estimates of representative yields.

Bloomberg representative yields are intended to be representative of yields on bonds within a similar ranking groups; i.e., Bloomberg representative yields on "A rated bonds" are intended to be representative of yields on A+, A and A- bonds. The Bloomberg representative yield on "A rated bonds" is determined so as to minimise the sum of squared deviations between actual observations on yields on A+, A and A- bonds and the Bloomberg representative yield on "A rated bonds". The Bloomberg representative yield on "A rated bonds" will tend to understate representative yields on bonds which actually have A-ratings (and will also tend to understate yields on A rated bonds if there are proportionally more A+ rated bonds in the sample)..

In the post June 2003 period examined in this study, all long dated bonds with a generic Bloomberg BBB rating are in fact BBB+ rated and no BBB or BBB- rated long dated bonds were used in the estimation of Bloomberg fitted yields on long dated "BBB rated bonds." The Bloomberg representative yield on long dated "BBB rated bonds" is determined so as to minimise the sum of squared deviations between actual yields on those BBB+ bonds and the Bloomberg representative yield on "BBB rated bonds". The Bloomberg representative yield on long dated "BBB rated bonds" will be an unbiased estimate of the yield on long dated BBB+ rated bonds. Table 3 reports the average difference between actual yields and Bloomberg representative yields.

Table 3
Average Differences between Actual Yields and Bloomberg Representative Yields over the period 30 June 2003 through 10 May 2005 by Bond

Bond	CFS Gandel 2010	Westfield	SPI Electricity	Tabcorp	Snowy	Stockland	CFS Gandel 2014
Average years to maturity[7]	6.2	6.5	6.8	6.7	8.7	8.9	9.8
Credit Rating	A	A*	A-	BBB+	BBB+	A-	A
Average excess to Bloomberg b.p.	1.12	3.02	7.82	–1.91	1.25	4.10	–4.58

*The Westfield bonds were downgraded to A- on 7 February 2005.

When estimated relative to Bloomberg fair yields there are both positive and negative average differences and those averages are much smaller in absolute value than the average differences between actual and CBASpectrum fair yields reported in Table 2.

[7] Average years to maturity is calculated as the average of the bond's time to maturity with the average calculated over each date between:

- 30 June 2003 or the date on which CBASpectrum first began to report data on the bonds (whichever comes later); and
- 10 May 2005 or the date at which they bond first has less than 6 years to maturity (whichever comes earlier).

5 FUNCTIONAL FORM

A second source of difference between the Bloomberg and CBASpectrum estimation techniques, relating to functional forms, may also be relevant to regulators when assessing the relative usefulness of CBASpectrum and Bloomberg.

5.1 Bloomberg's Functional Form

Bloomberg imposes only a very weak functional form on its estimates of 'fair'/'representative' yields on corporate debt. The Bloomberg estimation technique sets the yield curve for each credit rating relying solely on actual observations of bonds with the same credit rating. In doing so, it places no restrictions on the relation between the spread on bonds of a given rating and the time to maturity of those bonds, or on the relation between the spread on a bond of a given maturity and rating and the spread on a bond of the same maturity but different rating.

For each credit rating, Bloomberg nominates a number of predetermined maturity points on the yield curve (3 and 6 months, 1, 2, 3 4, 5, 7, 8, 9, 10, 15, and 20 years – or fewer if there are limited long dated observations). Bloomberg then estimates the yields to maturity on the set of bonds that would both sell at par and have maturity dates exactly equal to the predetermined maturity points. The estimation procedure minimises the sum of squared deviations between actual observed yields and fair yields on bonds, assuming that the fair yields on bonds selling at par with maturity dates between two nominated maturity points are determined from a straight line joining the fair yields on the two immediately surrounding bonds with maturities equal to the predetermined maturity points.

As such, there is no predetermined mathematical relationship (functional form) linking the values on the yield curve at each predetermined point. This means that the yields at those predetermined points are free to take any values that best fit the data – with the only constraint that these points are joined by a straight line. For example, if the best fit of the actual data on bonds of a given rating is that the yield curve rises, then falls and then rises again as maturity is increased then Bloomberg will fit a curve that rises, falls, then rises again. And the best fit of actual data on bonds of a different rating may be quite different again.

In summary, the Bloomberg estimation technique provides considerable flexibility to estimate:

- a variety of shapes for the yield curve; and
- different shapes for yield curves of different credit ratings.

5.2 CBASpectrum's Functional Form

The CBASpectrum technique imposes a functional form that, amongst other things, forces the fair/representative yield curves to have similar shapes for each credit rating. The effect of this is that the estimated shape of the yield curve for high rated bonds, for which there are many observations, affects the shape of the yield curve for low rated bonds, for which there are few observations.

The functional form imposed by CBASpectrum also forces, for any given maturity, the credit spread associated with a particular credit rating to be expressed as:

$$\begin{aligned}\text{Credit spread at each maturity} &= [\text{a term that depends on the bond's maturity}] \times [\text{a term that depends on the bond's rating}] \\ &= [\text{a term that depends on the bond's maturity}] \times e^{(a \times \text{CR} + b \times \text{CR}^2 + c \times \text{CR}^3)},\end{aligned}$$

where the term that depends on the bond's maturity describes how the spread increases with maturity for all bonds irrespective of their credit rating,

a, b and c are constants that are independent of the credit rating (CR), and

CR = 1 for AAA rated debt
= 2 for AA+ rated debt
= 3 for AA rated debt
= 4 for AA- rated debt
etc.

This functional form imposes a number of restrictions on the final estimated credit spread. For example, the credit spread for each credit rating is always a constant proportion of the credit spread for other credit ratings of the same maturity; ie, it does not allow the ratio of the credit spreads of different credit ratings to vary with maturity.[8] This restriction means that changes in maturity must have identical proportional impacts on credit spreads for all bonds – irrespective of their credit rating.

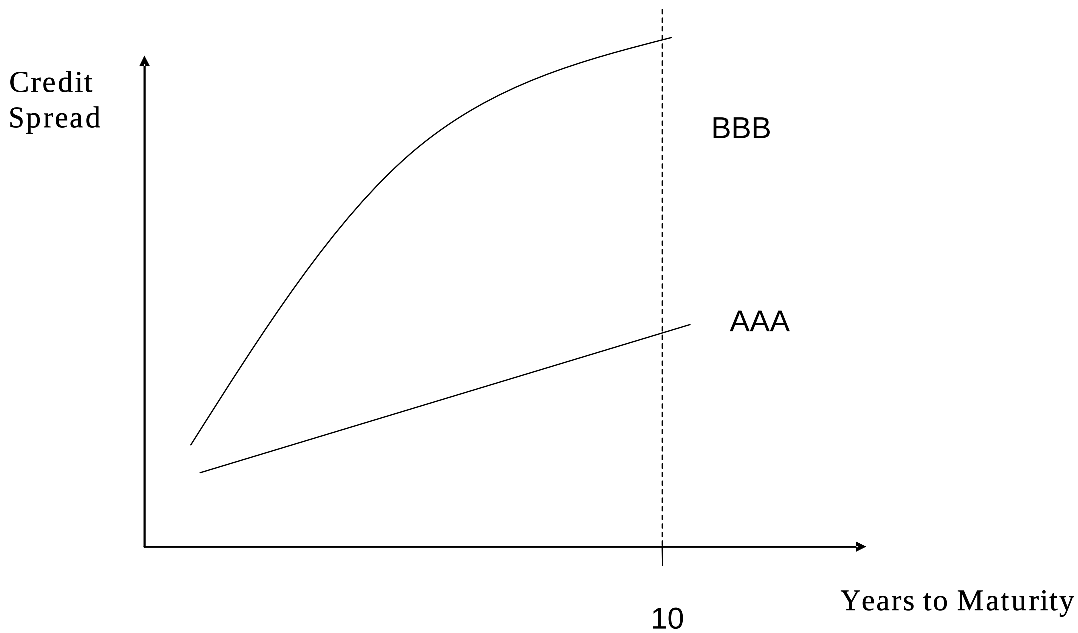
We are unaware of any empirical or theoretical reason to believe that such a relationship actually exists between spreads on corporate bonds. In fact, the seminal Merton (1974) theoretical work on the relation between yield spreads and maturity concludes that even when yield spreads on high-rated corporate bonds are increasing with maturity, yield spreads on low-rated corporate bonds may be humped shaped or decreasing with maturity.[9] Different shapes for the term structure of yields spreads on low versus high-grade bonds have been documented in Johnson (1967), Sarig and Warga (1989), Elton, Gruber, Agrawal

[8] For example, if the estimated credit spread on a BBB rated bond with one year to maturity is twice that estimated for a AAA rated bond with one year to maturity then a BBB rated bond with 10 years to maturity will also have twice the credit spread of a AAA rated bond with 10 years to maturity.

[9] Merton, Robert C. , 1974, "On the pricing of corporate debt: The risk structure of interest rates", ***Journal of Finance*** 29, pp. 449-470.

and Mann (2001), and Träuck, Laub and Rachev (2004). These researchers have examined data from different time periods, different industries and different countries.[10] In each case, the authors concluded that the relation between spreads and maturity does depend on the credit rating of the set of bonds being examined.

As an example of what can happen when the ratio of the credit spreads on bonds with different credit ratings does vary with maturity, imagine that bond traders believe that default risk increases proportionally faster with maturity the lower the bond's credit rating. In this case, the market credit spread on BBB versus AAA bonds may look something like that described in the graph below (ie, spreads may be very similar for maturities in the next few months but the BBB credit spread increases to triple the AAA credit spread at 10 years to maturity).



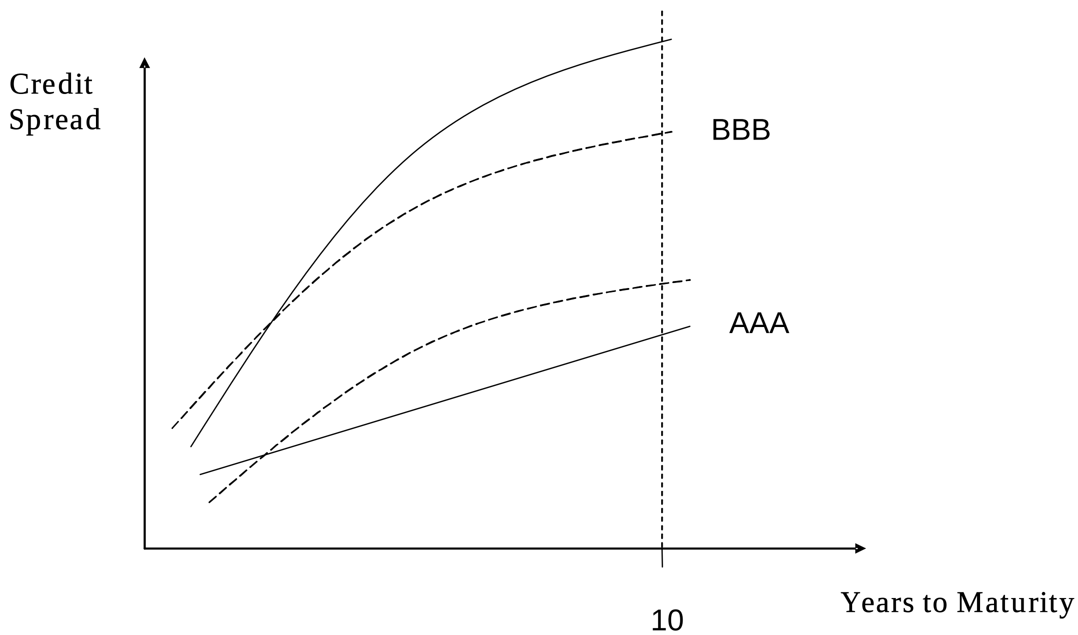
[10] Johnson, Ramon E., 1967, "Term structures of corporate bond yields as a function of risk of default," ***Journal of Finance*** 22, pp. 318—21.

Sarig, Oded and Arthur Warga, 1989, "Some empirical estimates of the risk structure of interest rates," ***Journal of Finance*** 44, pp. 1351—1360.

Elton, Edwin J., Martin J. Gruber, Deepak Agrawal and Christopher Mann, 2001, "Explaining the rate spread on corporate bonds," ***Journal of Finance*** 56, pp. 247—277.

Träuck, Stefan, Matthias Laub and Svetlozar T. Rachev, 2004, "The term structure of credit spreads and credit default swaps - an empirical investigation," Universität Karlsruhe Working Paper.

However, CBASpectrum's functional form can't reflect this hypothetical state of the market. The best 'fit' using CBASpectrum's functional form would involve shifting up/down the short/long maturity yield on BBB bonds to make this curve look more like the AAA curve (and ***vice versa*** for the AAA curve). This is depicted in the graph below with the dotted lines illustrating the type of impact on estimated credit spreads using a functional form that, like CBASpectrum, forces AAA and BBB credit spreads to have the same shape.



As can be seen, in this example forcing the yield curves to have the same shape underestimates yields on long dated BBB bonds but overestimates yields on long dated AAA bonds. The opposite may be true if, in reality, the ratio of BBB to AAA credit spreads actually fell as maturity increased. There are numerous other more subtle restrictions imposed by CBASpectrum's chosen functional form which we do not discuss in detail in this report. [11]

Imposing a functional form on the data, such as is applied by CBASpectrum, can be appropriate if there are reasons to believe that the functional form is correct – even if the data does not independently support that view. For example, it may be that the particular functional form has been tested against other data sets, say corporate bond yields in other countries, and has been found to accurately reflect relationships within those data sets. Before using CBASpectrum to set the regulated cost of debt, regulators should satisfy

11 For example, one of the mathematical properties of the cubic polynomial in the exponent of the CBASpectrum fitted credit spread is that it is only possible to have two 'points of inflexion' on the curve. (A point of inflexion exists where the second derivative of the function is zero (ie, the rate of change of the rate of change is zero)). This means that it is incapable of accurately describing a relationship with more than two points of inflexion (as would be the case if, for example, the relationship between credit ratings and credit spreads (at any given maturity) was stepwise with more than 2 steps).

themselves that there is a strong rationale for forcing the data to fit the functional form chosen by CBASpectrum. We are unaware of any such rationale for imposing the functional form chosen by CBASpectrum.

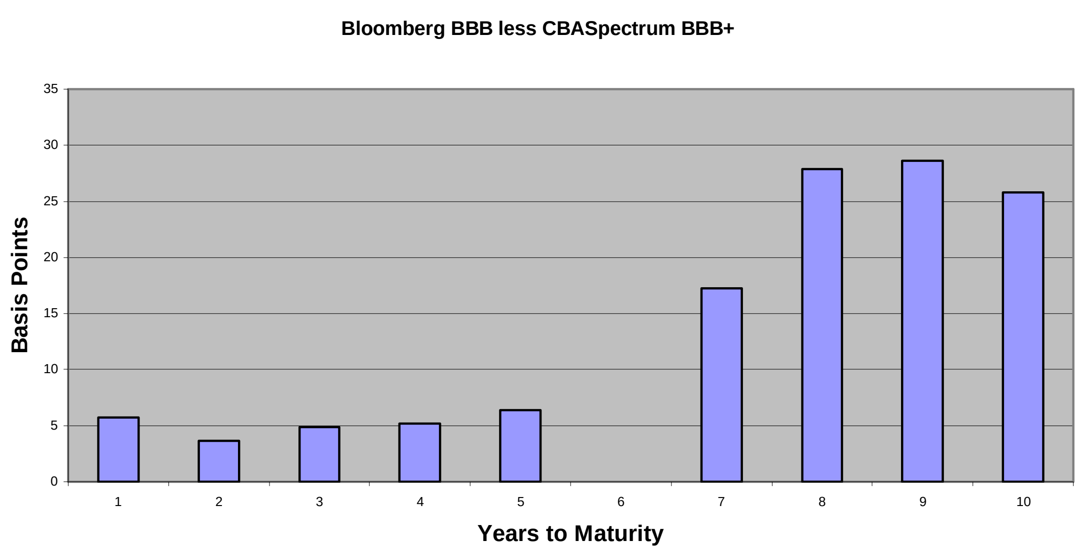
In the absence of any such rationale we believe that a more flexible functional form (ie, one that minimises the number of ***a priori*** judgments which the data is forced to fit) is superior. The Bloomberg functional form is more flexible than the CBASpectrum functional form and, hence, this flexibility is to be preferred - unless there is evidence that the restrictions imposed by CBASpectrum's estimation technique are justified by some other information set.

6 HISTORICAL DIFFERENCES BETWEEN BLOOMBERG AND CBASpectrum

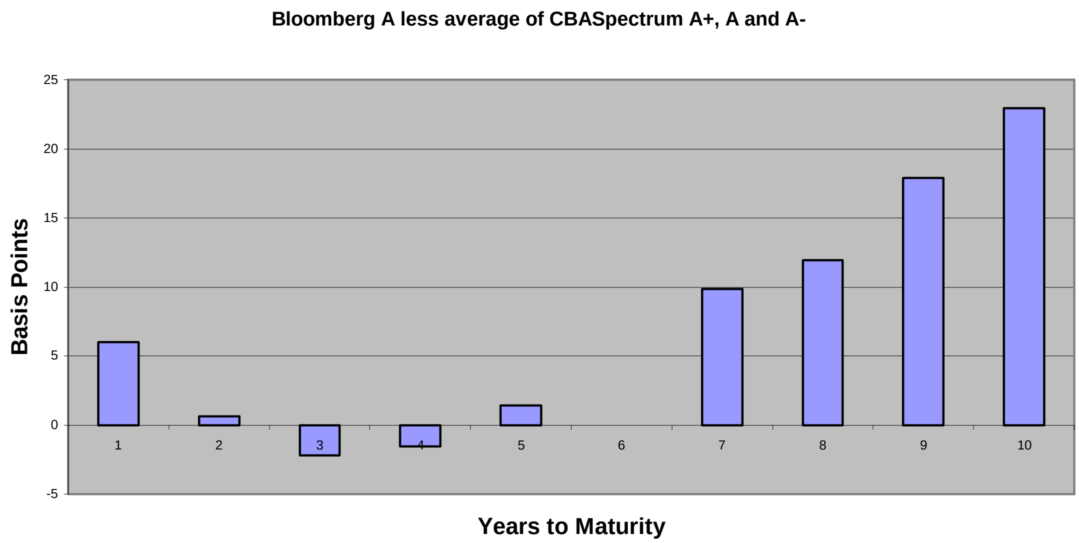
It is useful to ask whether the historical difference between Bloomberg and CBASpectrum estimates of fair yield is consistent with our own assessment of the likely underestimate in CBASpectrum for 10 year low rated bonds. Doing so will act as a 'check' on our own empirical analysis in section 3. If it is the case that Bloomberg estimates tend to be greater than CBASpectrum estimates by around 25 basis points (ie, our estimate of the likely underestimate on 10 year bonds) then this lends support to our own analysis. Symmetrically, it would lend support to the view that regulators should rely more heavily on Bloomberg estimates of fair yields.

Recall that the Bloomberg representative yield on long dated "BBB rated bonds" will be an unbiased estimate of the yield on a long dated BBB+ rated bond. The Figure below shows the average difference between Bloomberg representative yields on BBB bonds and CBASpectrum fair yields on BBB+ bonds for the period 30 June 2003 through 10 May 2005. Because of the absence of bonds whose yields could be used to determine a 10 year estimate, Bloomberg does not report a 10 year representative yield for BBB bonds after 20 October 2004. The average difference between Bloomberg and CBASpectrum yields on 10 year bonds is therefore calculated over the period 30 June 2003 through 20 October 2004.

Consistent with the results reported in the preceding sections, CBASpectrum fair yields understate the actual yields on long dated low rated bonds as reflected in Bloomberg representative yields. Over the period 30 June 2003 through 20 October 2004, CBASpectrum fair yields on 10 year BBB+ bonds understate Bloomberg representative yields by 25.76 basis points on average.



A similar difference between CBASpectrum fair yields on A+, A and A- bonds and actual yields on these bonds is observed if one compares an equal-weighted average of CBASpectrum fair yields on A+, A and A- bonds to the Bloomberg representative yield on "A rated bonds". This difference is depicted in the Figure below.



Again we see that CBASpectrum fair yields understate the actual yields on long dated low rated bonds. Over the period 30 June 2003 through 10 May 2005, an equal-weighted combination of CBASpectrum fair yields on 10 year A+, A and A- bonds understates the Bloomberg representative yield on A bonds by 22.93 basis points on average.

7 CONCLUSION

The CBASpectrum estimation procedure does not determine the best fit to the available data. The CBASpectrum estimation procedure is such that CBASpectrum estimated yields are expected to be, and in practice are, on average, less than actual yields for long-dated and low-rated bonds. Between 30 June 2003 and 10 May 2005, actual yields on Australian bonds with more than 6 years to maturity and ratings of A or below have averaged 17.1 basis points higher than the CBASpectrum estimated yields on such bonds. For bonds with more than 8 years to maturity and ratings of A or below, the difference has averaged 22.2 basis points.

We consider that, on the basis of the data examined in this report, the most appropriate adjustment to CBASpectrum estimates of yields on low rated (A and below) 10 year bonds is to add 25.6 basis points.

Alternatively, regulators may also wish to have regard to Bloomberg estimates of fair yields which for BBB 10 year bonds have tended to be 25.8 basis points above CBASpectrum estimates of fair yields.[12]

Finally, regulators may wish to engage in a consultation process in order to develop a 'tailor made' and transparent estimation technique to derive unbiased estimates of the cost of debt on long dated low rated corporate bonds.

[12] Recall that when there are no long dated bonds with rating of BBB or below, but there are long dated BBB+ bonds, then Bloomberg representative yields on long dated "BBB rated bonds" will be descriptive of yields on BBB+ bonds. However, Bloomberg will not estimate a 10 year yield on BBB+ bonds if there is no data on bonds with maturities in the relevant maturity range. In this case, some adjustment to Bloomberg 9 year yields would have to be made – one simple adjustment would be to add to the Bloomberg yield on a 9 year BBB bond an amount equal to the yield spread between 10 and 9 year CGS bonds.

APPENDIX A

SIMULATION OF THE DIFFERENCE BETWEEN ACTUAL CREDIT SPREADS AND CBASPECTRUM ESTIMATES OF CREDIT SPREADS

REPORT BY BRUCE DAVID GRUNDY*

Date of this report: 19 May 2005

This report simulates the difference between actual credit spreads and CBASpectrum estimates of credit spreads.

CBASprectrum estimated fair value yields taken the form:

$$\text{Fair Value Yield} = \text{CGS Yield} + \text{Spread},$$
$$\text{where Spread} = [\text{a term that depends on the bond's maturity}] \times [\text{a term that depends on the bond's rating}].$$

Without loss of generality, consider only two ratings (AAA and BBB) and two maturities (1 year and 10 year). CBASpectrum's estimation procedure is such that the two terms in square brackets above enter the determination of credit spreads in a multiplicative manner. Thus

$$\frac{\text{Average Spread on 1-year BBB bonds}}{\text{Average Spread on 1-year AAA bonds}} = \frac{\text{Average Spread on 10-year BBB bonds}}{\text{Average Spread on 10-year AAA bonds}}.$$

Suppose that observed bond yields satisfy this property of the CBASpectrum model perfectly.

The CBASpectrum technique will in this simple case (of 2 ratings classes and 2 maturity dates) mean that there are 3 parameters to be estimated: ***X***, ***Y*** and ***R.***

X = estimated spread on AAA bonds at 1 year

* Bruce Grundy is a Professor of Finance at the Melbourne Business School. This report has been prepared in my capacity as an independent consultant and does not reflect the views of my employer.

Y = estimated spread on AAA bonds at 10 years

R = ratio of estimated spread on BBB bonds to the estimated spread on AAA bonds.

X and ***Y*** determine the estimated spreads of AAA-rated bonds. ***R*** then multiplicatively determines the spreads of BBB bonds.

Within CBASpectrum's estimation technique a penalty is imposed on:

a) squared deviations between CBASpectrum estimates and actual observations (ie, the standard statistical technique);

b) plus the squared CBASpectrum estimate of the 'fair value' credit spread on AAA bonds at 10 years plus the squared difference between the estimated credit spread on BBB bonds at 10 years and the estimated credit spread on AAA bonds at 10 years;

c) plus the squared estimated spread on 1-year BBB bonds.

Let ***i*** index observations on the ***N*** observed bond yields. Having first estimated the CGS (Commonwealth Government Security) term structure, the CBASpectrum technique will in this setting solve the following problem:

$$Min_{X,Y,R} \sum_{i=1}^{N} \begin{pmatrix} [\text{CGS yield on a bond of } i\text{'s maturity} + \text{observed spread on a bond of } i\text{'s rating and maturity}] \\ -[\text{CGS yield on a bond of } i\text{'s maturity} + \text{estimated spread on a bond of } i\text{'s rating and maturity}] \end{pmatrix}^2$$
$$+([\text{CGS 1-year yield} + \text{estimated spread on 1-year BBB bonds}] - [\text{CGS 1-year yield}])^2$$
$$+([\text{CGS 10-year yield} + \text{estimated spread on 10-year AAA bonds}] - [\text{CGS 10-year yield}])^2$$
$$+([\text{CGS 10-year yield} + \text{estimated spread on 10-year BBB bonds}] - [\text{CGS 10-year yield + estimated spread on 10-year AAA bonds}])^2 .$$

$$Min_{X,Y,R} \sum_{i=1}^{N} ([\text{observed spread on a bond of } i\text{'s rating and maturity}] - [\text{estimated spread on a bond of } i\text{'s rating and maturity}])^2$$
$$+([\text{estimated spread on 1-year BBB bonds}])^2$$
$$+([\text{estimated spread on 10-year AAA bonds}])^2$$
$$+([\text{estimated spread on 10-year BBB bonds}] - [\text{ estimated spread on 10-year AAA bonds}])^2 .$$

The penalty associated with the squared estimated credit spread on 1-year BBB bonds corresponds to the penalty referred to in part c) of page 4 of this Report. The penalty associated with the squared estimated spread on 10-year AAA bonds plus the squared difference in spreads on 10-year BBB and 10-year AAA bonds corresponds to the penalty referred to in part b) of page 4 of this Report .

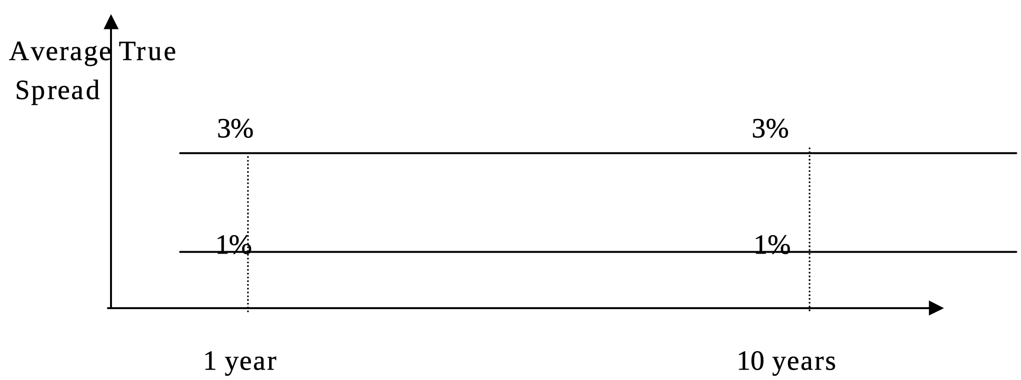
In this first simulation we consider a flat term structure of spreads. This does not imply that the term structure of CGS yields itself must be flat— the term structure of CGS yields may have any shape.

In the second (third) simulation, we consider a setting where spreads on long dated bonds of a given maturity exceed (are exceeded by) spreads on short-dated bonds of the same maturity.

In the first simulation consider a setting where:

- The average observed spread on 1 year AAA bonds is 1%, with actual spreads evenly distributed around 1%.
- The average observed spread on 1 year BBB bonds is 3%, with actual spreads evenly distributed around 3%. Note that the average spread on 1-year BBB is three times that on 1-year AAA bonds.
- The average observed spread on 10 year AAA bonds is 1%, with actual spreads evenly distributed around 1%.
- The average observed spread on 10 year BBB bonds is 3%, with actual spreads evenly distributed around 3%. Note that the average spread on 10-year BBB is also three times that on 1-year AAA bonds.

$$\frac{\text{Average Spread on 1-year BBB bonds}}{\text{Average Spread on 1-year AAA bonds}} = \frac{\text{Average Spread on 10-year BBB bonds}}{\text{Average Spread on 10-year AAA bonds}} = 3.$$



In the absence of the penalties in the CBASpectrum estimation procedure, the estimated parameters values would be ***X*** = 1%, ***Y*** = 1% and ***R*** = 3; i.e., in the absence of the penalties the CBASpectrum procedure will in this setting produce fair value yield estimates that are unbiased.

In the presence of the penalties, the difference between observed and estimated fair value yields will depend on the importance of the penalties relative to the observed data in the minimization problem. There are many more short-dated bonds than long dated bonds in the CBASpectrum database. There are also many more bonds with high ratings than with low ratings in the CBASpectrum database. To capture the features of the actual empirical distribution of observations in the CBASpectrum database, we consider a simulation that involves 100 observations on AAA-rated bonds with 1 year to maturity, 50 observations on AAA-rated bonds with 10 years to maturity, 50 observations on BBB-rated bonds with 1 year to maturity and 5 observations on BBB-rated bonds with 10 years to maturity.

Applying the CBASpectrum estimation technique with the penalties it imposes to the estimation of spreads in this dataset produces the following estimates of credit spreads:

X **=** 1.0247% = estimated spread on 1 year AAA bonds.
Y **=** 0.9753% = estimated spread on 10 year AAA bonds.
R **=** 2.856041 = estimated ratio of BBB spreads to AAA spreads.

Thus we have:

	# of Observations	True Average Spread	Estimated Spread	Difference
AAA 1	100	1.00%	1.025%	-0.025%
AAA 10	50	1.00%	0.975%	0.025%
BBB 1	50	3.00%	2.927%	0.073%
BBB 10	5	3.00%	2.786%	0.214%

Averaged across all observations, the average value of the Spread to CBA Spectrum is 0.017% (i.e., 1.7 basis points) in this analysis with a flat term structure of credit spreads.

Thus when the term structure of true average spreads is flat and we consider a distribution of observations on each rating/maturity class like the empirical distribution across ratings/maturity classes in the CBASpectrum database, the following results are obtained:

i) CBASpectrum estimated spreads are, on average, less than actual spreads.

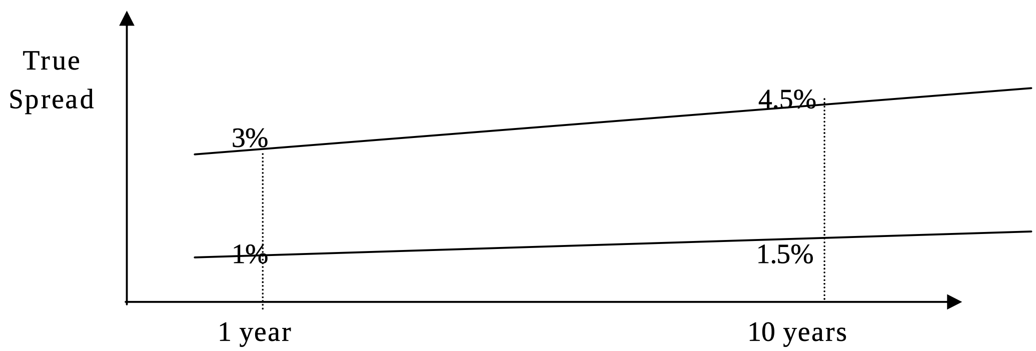
ii) For a given rating, the excess of actual spreads over CBASpectrum estimated spreads is larger for longer maturity bonds than for shorter maturity bonds.

iii) For a given maturity, the excess of actual spreads over CBASpectrum estimated spreads is larger for lower quality bonds than for higher quality bonds.

iv) The excess of actual spreads over CBASpectrum estimated spreads is largest for long dated, low rated bonds.

For completeness, our second simulation considers the case of an upward-sloping term structure of true average spreads. Suppose that:

- The average observed spread on 1 year AAA bonds is 1%, with actual spreads evenly distributed around 1%.
- The average observed spread on 1 year BBB bonds is 3%, with actual spreads evenly distributed around 3%. Note that the average spread on 1-year BBB is three times that on 1-year AAA bonds.
- The average observed spread on 10 year AAA bonds is 1.5%, with actual spreads evenly distributed around 1.5%.
- The average observed spread on 10 year BBB bonds is 4.5%, with actual spreads evenly distributed around 4.5%. Note that the average spread on 10-year BBB is also three times that on 1-year AAA bonds.



In the absence of the penalties in the CBASpectrum estimation procedure, the estimated parameters values would be ***X*** = 1%, ***Y*** = 1.5% and ***R*** = 3; i.e., in the absence of the penalties the CBASpectrum procedure will in this setting produce fair value yield estimates that are unbiased.

In the presence of the penalties in the CBASpectrum estimation procedure, the CBASpectrum estimation procedure applied to this dataset will produce the following estimates of credit spreads:

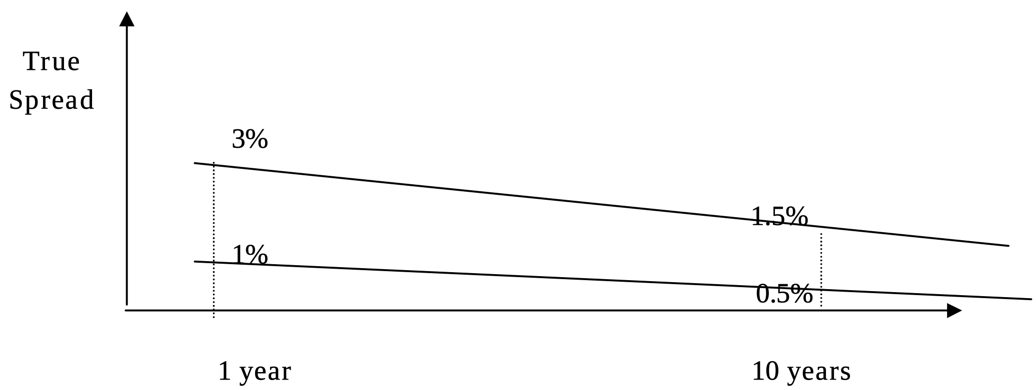
X = 1.0511% = estimated spread on 1 year AAA bonds.
Y = 1.4833% = estimated spread on 10 year AAA bonds.
R = 2.778186 = estimated ratio of BBB spreads to AAA spreads.

	# of Observations	True Average Spread	Estimated Spread	Difference
AAA 1	100	1.00%	1.051%	-0.051%
AAA 10	50	1.50%	1.483%	0.017%
BBB 1	50	3.00%	2.920%	0.080%
BBB 10	5	4.50%	4.121%	0.379%

Averaged across all observations, the average value of the Spread to CBA Spectrum is 0.008% (i.e., 0.8 basis points) in this analysis with an upward-sloping term structure of credit spreads.

Finally, all also for completeness, we consider the case of a downward-sloping term structure of true average spreads. Suppose that:

- The average observed spread on 1 year AAA bonds is 1%, with actual spreads evenly distributed around 1%.
- The average observed spread on 1 year BBB bonds is 3%, with actual spreads evenly distributed around 3%. Note that the average spread on 1-year BBB is three times that on 1-year AAA bonds.
- The average observed spread on 10 year AAA bonds is 0.5%, with actual spreads evenly distributed around 0.5%.
- The average observed spread on 10 year BBB bonds is 1.5%, with actual spreads evenly distributed around 1.5%. Note that the average spread on 10-year BBB is also three times that on 1-year AAA bonds.



In the absence of the penalties in the CBASpectrum estimation procedure, the estimated parameters values would be ***X*** = 1%, ***Y*** = 0.5% and ***R*** = 3; i.e., in the absence of the penalties the CBASpectrum procedure will in this setting produce fair value yield estimates that are unbiased.

In the presence of the penalties in the CBASpectrum estimation procedure, the CBASpectrum estimation technique applied to this dataset produces the following estimates of credit spreads:

X	=	1.02%	= estimated spread on 1 year AAA bonds.
Y	=	0.4869%	= estimated spread on 10 year AAA bonds.
R	=	2.878981	= estimated ratio of BBB spread to AAA spread.

	# of Observations	True Average Spread	Estimated Spread	Difference
AAA 1	100	1.00%	1.020%	-0.020%
AAA 10	50	0.50%	0.487%	0.013%
BBB 1	50	3.00%	2.937%	0.063%
BBB 10	5	1.50%	1.402%	0.098%

Averaged across all observations, the average value of the Spread to CBA Spectrum is 0.011% (i.e., 1.1 basis points) in this analysis with an upward-sloping term structure of credit spreads.

Conclusion

Whether the term structure of true average spreads is flat, upward-sloping or downward-sloping, when we consider a distribution of observations on each rating/maturity class like the empirical distribution across ratings/maturity classes in the CBASpectrum database, the following results hold:

i) CBASpectrum estimated spreads are, on average, less than actual spreads.

ii) For a given rating, the excess of actual spreads over CBASpectrum estimated spreads is larger for longer maturity bonds than for shorter maturity bonds.

iii) For a given maturity, the excess of actual spreads over CBASpectrum estimated spreads is larger for lower quality bonds than for higher quality bonds.

iv) The excess of actual spreads over CBASpectrum estimated spreads is largest for long dated, low rated bonds.

APPENDIX B

Estimating Credit Spreads on Long Term, Low Rated Bonds

A Report by

Kevin Davis[13]

May 27, 2005

1. In preparing this report, I have had access to a document entitled "Spectrum Re-Built Analytics".

2. The purpose of this report is to assess whether the CBA Spectrum approach to estimating fair yield spreads leads to a downward bias in the estimated "fair" spreads for long term (10 year) lower rated (A- or less) corporate bonds.

3. This report does not make any judgment on the general merits of the CBA Spectrum approach for the more general purpose (for which the fair spread estimates are likely to be primarily used) of estimating spreads for bonds at the shorter end of the yield curve. I note that the approach used is adopted as a method of overcoming estimation problems which arise because of the sparseness of observations of corporate bonds of various ratings and maturities. I do not have access to the CBA Spectrum database, and thus cannot address issues such as the number of corporate bonds of various maturities used in the estimation process, nor assess claims about the relationship between fitted and observed spreads.

4. I conclude that: There is a substantive argument that CBASpectrum's estimation technique creates a material downward bias in estimates of "fair spread" for long dated and low rated bonds

[13] Kevin Davis is a Professor of Finance in the Department of Finance, Faculty of Economics and Commerce, The University of Melbourne. This report has been prepared in my capacity as an independent consultant and does not reflect the views of my employer.

The CBA Spectrum Estimation Method

5. The documentation of the CBA Spectrum estimation method provided (entitled "Spectrum Re-Built Analytics") contains relatively little by way of explanation of the relationship between the code provided and underlying yield curve modeling and requires the reader to use some judgment in interpretation.

6. It is, however, clear that the approach involves several steps. First, a yield curve for CGS (Commonwealth Government Bonds) is estimated by applying the Nelson-Siegel formula for estimated (fitted) yields and minimizing the sum of squared differences between observed and estimated yields. Second, a "fair yield" function is estimated for corporate bonds of different maturity and ratings. The fair yield on any particular corporate bond is defined as the estimated CGS yield for the same maturity plus an estimated spread term (denoted as $e^{g(i)}$ in the documentation provided) which is modeled as depending on maturity and on rating. The mathematical form used for this spread term means that it can be written as involving a function of maturity multiplied by a function of rating.

7. To estimate the spread terms for all corporate bonds, the approach adopted is to minimize a function which equals the sum of three components. The first component (a) is the sum of squared differences between actual and estimated corporate yields. The second component (b) is the squared difference between the estimated yields on Government and BBB- bonds of one year maturity. The third component (c) is the sum of squared estimated differences in yields between successive ratings categories for bonds of ten year maturity.

8. The first component (a) is a standard term. Used in isolation, the result would be a "best fit" between estimated and actual yields based on the criteria of minimizing the sum of squared differences. The latter two components can be interpreted as "penalty functions" which are included, presumably, to influence the estimated results in a fashion consistent with some undefined objective. They may be needed to offset unwanted effects on estimated values arising from the specific functional form assumed for the estimated spread term. Use of such an approach appears necessary because the small number of corporate bonds in various ratings / maturity groupings makes direct estimation of a fair spread for each grouping infeasible.

9. The second component (b) has the effect of reducing (ie biasing downwards) the estimated spread for BBB- bonds of one year maturity, and the approach can be interpreted as introducing into the estimation process an additional "phantom" BBB- one year maturity bond with a zero spread over the government yield curve. The third component (c) will have the effect of biasing downwards the estimated yield on low rated 10 year corporate bonds (ie giving estimated spreads which are below those which would be calculated if there were sufficient observations in that grouping to enable the spread to be estimated directly). It can be interpreted as involving the introduction of "phantom" bonds with particular spread characteristics into the estimation process. The complexity of the CBA Spectrum analytics makes mathematical proof of the bias a complicated exercise, but it can be relatively easily demonstrated in a simplified case. This is done in the appendix to this report, where it is shown that the bias will be larger when there are relatively small numbers of bonds in the long term, low rating category.

10. Conclusions:

- There is a substantive argument that CBASpectrum's estimation technique creates a material downward bias in estimates of "fair spread" for long dated and low rated bonds.

Appendix: A simplified example

This appendix provides a mathematical demonstration of how, in a simple example, a minimization process involving "penalty functions" such as that used in CBA Spectrum can lead to biased estimates of spreads on long dated low rated bonds. Note that in the CBA Spectrum approach, the estimated spread terms such as $\hat{m}_B^1$ are functions involving six parameters, and the estimation process involves minimizing with respect to those parameters. Here, the minimization is done directly using the spread terms $\hat{m}_B^1$ etc. to keep the mathematics tractable.

Let the return on corporate bond ***i*** be denoted by r_i and expressed as:

$$r_i = g_i + m_i$$

where g_i is the yield on a government bond of the same maturity and m_i is the observed spread for bond ***i.***

Suppose there are:

n_1 corporate bonds of B rating, 1 year maturity - spreads denoted by $m_{B,i}^1$, $i=1\ldots n_1$.

n_2 corporate bonds of A rating, 1 year maturity- spreads denoted by $m_{A,i}^1$, $i=1\ldots n_2$.

n_3 corporate bonds of B rating, 10 year maturity- spreads denoted by $m_{B,i}^{10}$, $i=1\ldots n_3$.

n_4 corporate bonds of A rating, 10 year maturity- spreads denoted by $m_{A,i}^{10}$, $i=1\ldots n_4$.

The CBA Spectrum approach for this simplified example involves choosing a spread value for each group $\hat{m}_B^1;\hat{m}_A^1;\hat{m}_B^{10};\hat{m}_A^{10}$ to minimize

$$L = \sum_{i=1}^{n1}(m_{B,i}^1 - \hat{m}_B^1)^2 + \sum_{i=1}^{n2}(m_{A,i}^1 - \hat{m}_A^1)^2 + \sum_{i=1}^{n3}(m_{B,i}^{10} - \hat{m}_B^{10})^2 + \sum_{i=1}^{n4}(m_{A,i}^{10} - \hat{m}_A^{10})^2$$
$$+(\hat{m}_B^1)^2 + (\hat{m}_A^{10})^2 + (\hat{m}_B^{10} - \hat{m}_A^{10})^2$$

Minimizing L with respect to $\hat{m}_B^1;\hat{m}_A^1;\hat{m}_B^{10};\hat{m}_A^{10}$ gives:

$$\frac{\partial L}{\partial \hat{m}_B^1} = -2\sum_{i=1}^{n1}(m_{B,i}^1 - \hat{m}_B^1) + 2\hat{m}_B^1 = 0$$
$$\frac{\partial L}{\partial \hat{m}_A^1} = -2\sum_{i=1}^{n2}(m_{A,i}^1 - \hat{m}_A^1) = 0$$
$$\frac{\partial L}{\partial \hat{m}_B^{10}} = -2\sum_{i=1}^{n3}(m_{B,i}^{10} - \hat{m}_B^{10}) + 2(\hat{m}_B^{10} - \hat{m}_A^{10}) = 0$$
$$\frac{\partial L}{\partial \hat{m}_A^{10}} = -2\sum_{i=1}^{n4}(m_{A,i}^{10} - \hat{m}_A^{10}) + 2\hat{m}_A^{10} - 2(\hat{m}_B^{10} - \hat{m}_A^{10}) = 0$$

Rewriting these equations and using mean notation $\overline{m}$ as appropriate gives

$$\hat{m}_A^1 = \sum_{i=1}^{n2} m_{A,i}^1 /(n_2) = \overline{m}_A^1$$
$$\hat{m}_B^1 = \sum_{i=1}^{n1} m_{B,i}^1 /(n_1 + 1) = \overline{m}_B^1(n_1 /(n_1 + 1))$$
$$(n_3 + 1)\hat{m}_B^{10} - \hat{m}_A^{10} = n_3\overline{m}_B^{10}$$
$$(n_4 + 2)\hat{m}_A^{10} - \hat{m}_B^{10} = n_4\overline{m}_A^{10}$$

It can be seen from the first two equations that

- $\hat{m}_B^1$ is biased downwards
- $\hat{m}_A^1$ is unaffected.

Solving the latter two equations for $\hat{m}_B^{10}$ gives;

$$\hat{m}_B^{10} = \overline{m}_B^{10} - \frac{n_4(\overline{m}_B^{10} - \overline{m}_A^{10}) + \overline{m}_B^{10}}{n_3 n_4 + n_4 + 2n_3 + 1} < \overline{m}_B^{10}$$

- $\hat{m}_B^{10}$ is biased downwards relative to $\overline{m}_B^{10}$.

It can be seen that the downward bias in $\hat{m}_B^{10}$ depends upon the number of corporate bonds in the 10 year category (***n_3*** and ***n_4***).

If there are many bonds then the bias will be small (since the term in the denominator involving ***$n_3 n_4$*** will be very large). The effect of the "penalty function" represented by the last two terms of the minimization in this case tends to be to equalize the differences between the spreads on bonds of successive ratings categories, without significantly affecting the estimated spread for the lowest rated bond.

If there are few bonds in this category, the bias will be larger (and in the limit $\hat{m}_B^{10}$ would approach 0 as the number of bonds in the ten year category approach zero). The effect of the "penalty function" represented by the last two terms of the minimization in this case tends to be to reduce the estimated spread for the lowest rated bond.