

Robust Regression Techniques

A report for DBP

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Project Team

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Executive Summary

This report has been prepared for DBP by NERA Economic Consulting (NERA). The Economic Regulation Authority (ERA), in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, advocates the use of robust regression techniques to estimate the equity beta of a regulated energy utility.¹ DBP has asked NERA to assess the costs and benefits of using robust regression techniques to estimate the equity beta of a regulated energy utility.

It is well known that a benefit to using robust regression estimates is that they are less sensitive to extreme observations and so typically less variable than their ordinary least squares (OLS) counterparts. It is less well known that a cost associated with the use of robust regression estimates is that the estimates can be biased. DBP has asked NERA:

- to show how a bias associated with robust regression estimates can arise.

The ERA, in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, suggests that one choose between least absolute deviations (LAD) and OLS estimators on the basis of goodness-of-fit tests.² The ERA in the appendices also examines the behaviour of OLS and robust regression estimators using bootstrap simulations.³ DBP has asked NERA:

- to use bootstrap simulations like those that the ERA employs to examine the behaviour of estimators constructed using a goodness-of-fit screening strategy of the kind that the ERA suggests one adopt.

If the benefits of using robust regression techniques outweigh the costs and the market for academic research is efficient, then one should expect to find evidence of the frequent use of the techniques in published work. Thus DBP has also asked NERA:

- to review the finance literature to determine the extent to which robust estimation techniques are used in research.

Bias

Kennedy (1979) defines a robust estimator to be:⁴

‘one whose desirable properties are insensitive to departures from the assumptions under which it is derived.’

In the statistics literature, robust regression estimators are labelled ‘robust’ because they are relatively insensitive to extreme observations. Robust regression estimators may still retain

¹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 145-180.

² ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

³ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 163-180.

⁴ Kennedy, P., *A guide to econometrics*, MIT Press, 1979, page 24.

this desirable characteristic even when the assumptions under which they are derived do not hold. Departures from the assumptions under which robust regression estimators are derived, however, can lead to the estimators losing other important properties. In particular, departures from the assumptions under which robust regression estimators are derived can lead to the estimators losing the property of consistency. An estimator is said to be consistent if it converges in probability to the correct population value as the sample size grows. As Imbens and Wooldridge (2007) note:⁵

‘so-called “robust” estimators, which are intended to be insensitive to outliers or influential data, usually require symmetry of the error distribution for consistent estimation. Thus, they are not “robust” in the sense of delivering consistency under a wide range of assumptions.’

The ERA in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013 provides an analysis of the LAD estimator.⁶ As the ERA notes, the LAD estimators of the parameters of a regression will be maximum likelihood if the disturbance from the regression follows a Laplace distribution and so under this condition will be consistent. If, however, the disturbance from the regression does not follow a Laplace distribution – which is a symmetric distribution – the estimators need not be consistent. In particular, if the distribution of the disturbance is skewed, then the LAD estimators can be biased. In contrast, as Wooldridge notes:⁷

‘OLS produces unbiased and consistent estimators ... whether or not the error distribution is symmetric; symmetry does not appear among the Gauss-Markov assumptions.’

It is also true that, contrary to the assertion that the ERA (2013) makes, the Gauss-Markhov Theorem does not require the disturbance from a regression to be normally distributed.⁸⁹ We demonstrate using simulations that LAD estimators can be biased while OLS estimators are simultaneously unbiased when the distribution of the disturbance from a regression is skewed. We also use graphs to provide some intuition on how the bias that can be associated with the estimators can arise.

Screening strategy

The ERA, in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, suggests that one choose between least absolute deviations (LAD) and

⁵ Imbens, G.M. and J. Wooldridge, *Quantile methods*, NBER lecture notes, 2007, page 2.

Available at:

http://www.nber.org/WNE/lect_14_quantile.pdf

⁶ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 145-180.

⁷ Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 333.

⁸ The Gauss-Markhov Theorem states that under certain conditions OLS estimators will be Best Linear Unbiased, that is, they will have the smallest variance amongst all linear unbiased estimators.

⁹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, page 166.

Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 102.

OLS estimators on the basis of goodness-of-fit tests.¹⁰ The ERA in the appendices also provides the results of bootstrap simulations that examine the behaviour of OLS and robust regression estimates.¹¹ Surprisingly, the regulator does not take the opportunity of assessing a goodness-of-fit screening strategy like that it suggests one use.

We conduct bootstrap simulations like those that the ERA uses. We find that:

- the three robust regression techniques that the ERA employs typically provide biased estimates of beta whereas the OLS estimates exhibit no significant bias;
- the three robust regression techniques typically provide estimates of beta that are more precise;
- the tests that Puig and Stephens (2000) advocate one use can fail to detect departures from the Laplace null hypothesis;¹² and
- a screening strategy of using a Laplace goodness-of-fit test to determine whether to use LAD does not perform well – for four of the six stocks that we use and for one of the two portfolios that we employ OLS estimates display both less bias and a lower variability than estimates that use the screening strategy.

Robust regression usage

If the benefits of using robust regression techniques exceed the costs and the market for academic research is efficient, then one should expect to find evidence of the frequent use of these techniques in published work.

We conduct keyword searches of the four major finance journals as a way of discovering how frequently robust regression techniques are used in high quality research relative to OLS.

The four journals that we select are the Journal of Finance, the Journal of Financial Economics, the Journal of Financial and Quantitative Analysis and the Review of Financial Studies. These are the four finance journals included in the list of 45 journals used by the Financial Times in compiling its business school research rankings.¹³ They are also the four journals which a recent study of finance journal rankings that Currie and Pandher (2010) conduct rate most highly in terms of their quality.¹⁴

We find that in these four journals there are relatively few references to robust regression techniques. We search for references to the phrase ‘ordinary least squares’ and references to nine phrases that concern three different robust regression techniques. Across the Journal of

¹⁰ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

¹¹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 163-180.

¹² Puig, P. and M.A. Stephens, *Tests of fit for the Laplace distribution with applications*, Technometrics, 2000, pages 417-424.

¹³ <http://www.ft.com/intl/cms/s/2/3405a512-5cbb-11e1-8f1f-00144feabdc0.html#axzz3LBbELIWS>

¹⁴ Currie, R.R. and G. S. Pandher, *Finance journal rankings and tiers: An active scholar assessment methodology*, Journal of Banking and Finance, 2011, pages 7-20.

Finance, the Journal of Financial Economics and the Review of Financial Studies – the journals that Currie and Pandher rate most highly in terms of quality – we find 1,449 references to the phrase ‘ordinary least squares’ and 75 references to the nine phrases that concern robust regression techniques. In the Journal of Financial and Quantitative Analysis we find surprisingly few references to any of the 10 phrases. We find 27 references to the phrase ‘ordinary least squares’ and 8 references to the nine phrases that concern robust regression techniques.

Thus the evidence that we provide strongly suggests that robust regression techniques are used infrequently in high quality finance research.

1. Introduction

This report has been prepared for DBP by NERA Economic Consulting (NERA). The Economic Regulation Authority (ERA), in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, advocates the use of robust regression techniques to estimate the equity beta of a regulated energy utility.¹⁵ DBP has asked NERA to assess the costs and benefits of using robust regression techniques to estimate the equity beta of a regulated energy utility.

It is well known that a benefit to using robust regression estimates is that they are less sensitive to extreme observations and so typically less variable than their ordinary least squares (OLS) counterparts. It is less well known that a cost associated with the use of robust regression estimates is that the estimates can be biased. DBP has asked NERA:

- to show how a bias associated with robust regression estimates can arise.

The ERA, in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, suggests that one choose between least absolute deviations (LAD) and OLS estimators on the basis of goodness-of-fit tests.¹⁶ The ERA in the appendices also examines the behaviour of OLS and robust regression estimators using bootstrap simulations.¹⁷ DBP has asked NERA:

- to use bootstrap simulations like those that the ERA employs to examine the behaviour of estimators constructed using a goodness-of-fit screening strategy of the kind that the ERA suggests one adopt.

If the benefits of using robust regression techniques outweigh the costs and the market for academic research is efficient, then one should expect to find evidence of the frequent use of the techniques in published work. Thus DBP has also asked NERA:

- to review the finance literature to determine the extent to which robust estimation techniques are used in research.

The remainder of this report is structured as follows:

- section 2 demonstrates using simulations that LAD estimators can be biased while OLS estimators are simultaneously unbiased when the distribution of the disturbance from a regression is skewed;
- section 3 presents the results of Gaussian and Laplace goodness-of-fit tests and OLS and robust regression estimates of the equity beta of a regulated energy utility;

¹⁵ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 145-180.

¹⁶ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

¹⁷ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 163-180.

- section 4 presents the results of bootstrap simulations that examine the behaviour of several robust regression estimates and a screening strategy of the kind that the ERA suggests one adopt;
- section 5 conducts keyword searches of the four major finance journals as a way of discovering how frequently robust regression techniques are used in high quality research relative to OLS; and
- section 6 offers conclusions.

In addition:

- Appendix A provides the terms of reference for this report;
- Appendix B provides a copy of the Federal Court of Australia's *Guidelines for Expert Witnesses in Proceedings in the Federal Court of Australia*; and
- Appendix C provides the curriculum vitae of the author of the report.

Statement of Credentials

This report has been prepared by **Simon Wheatley**.

Simon Wheatley is an Affiliated Industry Expert with NERA, and was until 2008 a Professor of Finance at the University of Melbourne. Since 2008, Simon has applied his finance expertise in investment management and consulting outside the university sector. Simon's interests and expertise are in individual portfolio choice theory, testing asset-pricing models and determining the extent to which returns are predictable. Prior to joining the University of Melbourne, Simon taught finance at the Universities of British Columbia, Chicago, New South Wales, Rochester and Washington.

In preparing this report, the author (herein after referred to as 'I' or 'my' or 'me') confirms that I have made all the inquiries that I believe are desirable and appropriate and that no matters of significance that I regard as relevant have, to my knowledge, been withheld from this report. I acknowledge that I have read, understood and complied with the Federal Court of Australia's *Practice Note CM 7, Expert Witnesses in Proceedings in the Federal Court of Australia*. I have been provided with a copy of the Federal Court of Australia's *Practice Note CM 7, Expert Witnesses in Proceedings in the Federal Court of Australia*, dated 4 June 2013, and my report has been prepared in accordance with those guidelines.

I have undertaken consultancy assignments for DBP in the past. However, I remain at arm's length, and as an independent consultant.

2. Theory

Kennedy (1979) defines a robust estimator to be:¹⁸

‘one whose desirable properties are insensitive to departures from the assumptions under which it is derived.’

In the statistics literature, robust regression estimators are labelled ‘robust’ because they are relatively insensitive to extreme observations. Robust regression estimators may still retain this desirable characteristic even when the assumptions under which they are derived do not hold. Departures from the assumptions under which robust regression estimators are derived, however, can lead to the estimators losing other important properties. In particular, departures from the assumptions under which robust regression estimators are derived can lead to the estimators losing the property of consistency. An estimator is said to be consistent if it converges in probability to the correct population value as the sample size grows. As Imbens and Wooldridge (2007) note:¹⁹

‘so-called “robust” estimators, which are intended to be insensitive to outliers or influential data, usually require symmetry of the error distribution for consistent estimation. Thus, they are not “robust” in the sense of delivering consistency under a wide range of assumptions.’

The ERA in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013 provides an analysis of the LAD estimator.²⁰ As the ERA notes, the LAD estimators of the parameters of a regression will be maximum likelihood if the disturbance from the regression follows a Laplace distribution and so under this condition will be consistent. If, however, the disturbance from the regression does not follow a Laplace distribution – which is a symmetric distribution – the estimators need not be consistent. In particular, if the distribution of the disturbance is skewed, then the LAD estimators can be biased. In contrast, as Wooldridge notes:²¹

‘OLS produces unbiased and consistent estimators ... whether or not the error distribution is symmetric; symmetry does not appear among the Gauss-Markov assumptions.’

We demonstrate using simulations that LAD estimators can be biased while OLS estimators are simultaneously unbiased when the distribution of the disturbance from a regression is skewed. We also use graphs to provide some intuition on how the bias that can be associated with the estimators can arise.

¹⁸ Kennedy, P., *A guide to econometrics*, MIT Press, 1979, page 24.

¹⁹ Imbens, G.M. and J. Wooldridge, *Quantile methods*, NBER lecture notes, 2007, page 2.

Available at:

http://www.nber.org/WNE/lect_14_quantile.pdf

²⁰ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 145-180.

²¹ Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 333.

2.1. LAD versus OLS regression

Given a sample $\{y_t, x_t\}$, $t = 1, 2, \dots, T$, where y_t and x_t are random variables, the ordinary least squares (OLS) estimators of the parameters α and β in a linear model minimise the sum of squared errors:

$$\sum_{t=1}^T (y_t - \alpha - \beta x_t)^2 \quad (1)$$

In contrast, the LAD estimators of the parameters α and β in a linear model minimise the sum of the absolute values of the errors:

$$\sum_{t=1}^T |y_t - \alpha - \beta x_t| \quad (2)$$

It can be shown that OLS will fit the model:²²

$$E(y_t | x_t) = \alpha + \beta x_t \quad (3)$$

where $E(y_t | x_t)$ denotes the mean of y_t conditional on x_t , while LAD will fit the model:

$$\text{Med}(y_t | x_t) = \alpha + \beta x_t \quad (4)$$

where $\text{Med}(y_t | x_t)$ denotes the median of y_t conditional on x_t . Since the median is not affected by large changes in extreme observations, LAD estimates are less sensitive to outliers than are OLS estimates. As Wooldridge (2013) notes, however:²³

‘When LAD and OLS are applied to cases with asymmetric distributions, the estimated partial effect of (x_t) obtained from LAD can be very different from the partial effect obtained from OLS. But such a difference could just reflect the difference between the median and the mean and might not have anything to do with outliers.’

Similarly, as Imbens and Wooldridge (2007) emphasise:

‘LAD is much more resilient to changes in extreme values because, as a measure of central tendency, the median is much less sensitive than the mean to changes in

²² Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, pages 332-333.

²³ Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 333.

extreme values. But it does not follow that a large difference in OLS and LAD estimates means something is “wrong” with OLS.’

Note that equation (3) implies that:

$$y_t = \alpha + \beta x_t + \varepsilon_t \quad (5)$$

where $E(\varepsilon_t) = \text{Cov}(x_t, \varepsilon_t) = 0$. From (5), it follows that:

$$\beta = \text{Cov}(x_t, y_t) / \text{Var}(x_t) \quad (6)$$

If x_t represents the return to the market portfolio and y_t the return to the equity of a firm, then equation (6) provides the formula for the equity beta of the firm. In other words, if x_t represents the return to the market portfolio and y_t the return to the equity of a firm, then the OLS parameter β that appears in equation (3) will be the equity beta of the firm.

If the conditional median $\text{Med}(y_t | x_t)$ differs from the conditional mean $E(y_t | x_t)$ by at most a constant, then the parameter β in equation (4) will match the parameter β in equation (5). If, on the other hand, the conditional median $\text{Med}(y_t | x_t)$ differs from the conditional mean $E(y_t | x_t)$ by an amount that is not constant, then the parameter β in equation (4) need not match the parameter β in equation (5).

It follows that if x_t represents the return to the market portfolio and y_t the return to the equity of a firm, then the LAD parameter β that appears in equation (4) need not be the equity beta of the firm.

We now set about illustrating these ideas using some simple examples.

2.2. Examples

To illustrate the bias that can be associated with robust regression estimators of the slope coefficient in a regression, we use the non-central t distribution. We choose this distribution because Harvey and Siddique (1999) use the distribution to model skewness in returns.²⁴

Let u_t and v_t be independently distributed with

$$u_t \sim N(\lambda_t, 1) \quad (7)$$

and

$$v_t \sim \chi_q^2 \quad (8)$$

²⁴ Harvey, C.R. and A. Siddique, *Autoregressive conditional skewness*, Journal of Financial and Quantitative Analysis, 1999, pages 465-487.

Then the ratio

$$u_t / (v_t / q)^{1/2} \quad (9)$$

will be t distributed with q degrees of freedom and non-centrality parameter λ_t .²⁵ The distribution of the ratio will be skewed to the left – that is, it will exhibit negative skewness – if the non-centrality parameter $\lambda_t < 0$ while the distribution will be skewed to the right – it will exhibit positive skewness – if the non-centrality parameter $\lambda_t > 0$.

We generate data that satisfies the regression (5) with:²⁶

$$\alpha = 0, \quad \beta = 1, \quad x_t \sim N(0,1), \\ \varepsilon_t = u_t / (v_t / q)^{1/2} - E(u_t / (v_t / q)^{1/2}), \quad \lambda_t = \phi x_t, \quad q = 10 \quad (10)$$

If $\phi > 0$, then the distribution of the disturbance ε_t will be skewed to the left when $x_t < 0$, it will be symmetric when $x_t = 0$ and skewed to the right when $x_t > 0$. Thus if $\phi > 0$, then the conditional median $\text{Med}(y_t | x_t)$ will differ from the conditional mean $E(y_t | x_t)$ by an amount that is not constant. Thus we would expect LAD estimators of the regression parameter β to be biased.

If $\phi < 0$, then the distribution of the disturbance ε_t will be skewed to the right when $x_t < 0$, it will be symmetric when $x_t = 0$ and skewed to the left when $x_t > 0$. Thus if $\phi < 0$, then the conditional median $\text{Med}(y_t | x_t)$ will also differ from the conditional mean $E(y_t | x_t)$ by an amount that is not constant. Thus we would again expect LAD estimators of β to be biased.

Table 2.1 confirms these predictions. The table provides the results of 10,000 replications each of which uses 100 observations generated using the regression (5) and the assumptions (10). The table indicates that OLS estimates of β are unbiased while LAD estimates can be biased. If $\phi < 0$, LAD estimates are upwardly biased, if $\phi = 0$, LAD estimates are unbiased while if $\phi > 0$, LAD estimates are downwardly biased.

Intuition for how this bias arises can be provided by examining plots of y_t against x_t . Figure 2.1 provides a plot of y_t against x_t for a single sample of 10,000 observations when $\phi = 20$.

²⁵ Bain, L.J., *Moments of a noncentral t and noncentral F-distribution*, American Statistician, 1969, pages 33-34.

Hogben, D., R. S. Pinkham and M. B. Wilk, *The moments of the non-central t-distribution*, Biometrika, 1961, pages 465-468.

²⁶ We compute $E(u_t / (v_t / q)^{1/2})$ using results in Hogben, Pinkham and Wilk (1961) and Bain (1969).

Bain, L.J., *Moments of a noncentral t and noncentral F-distribution*, American Statistician, 1969, pages 33-34.

Hogben, D., R. S. Pinkham and M. B. Wilk, *The moments of the non-central t-distribution*, Biometrika, 1961, pages 465-468.

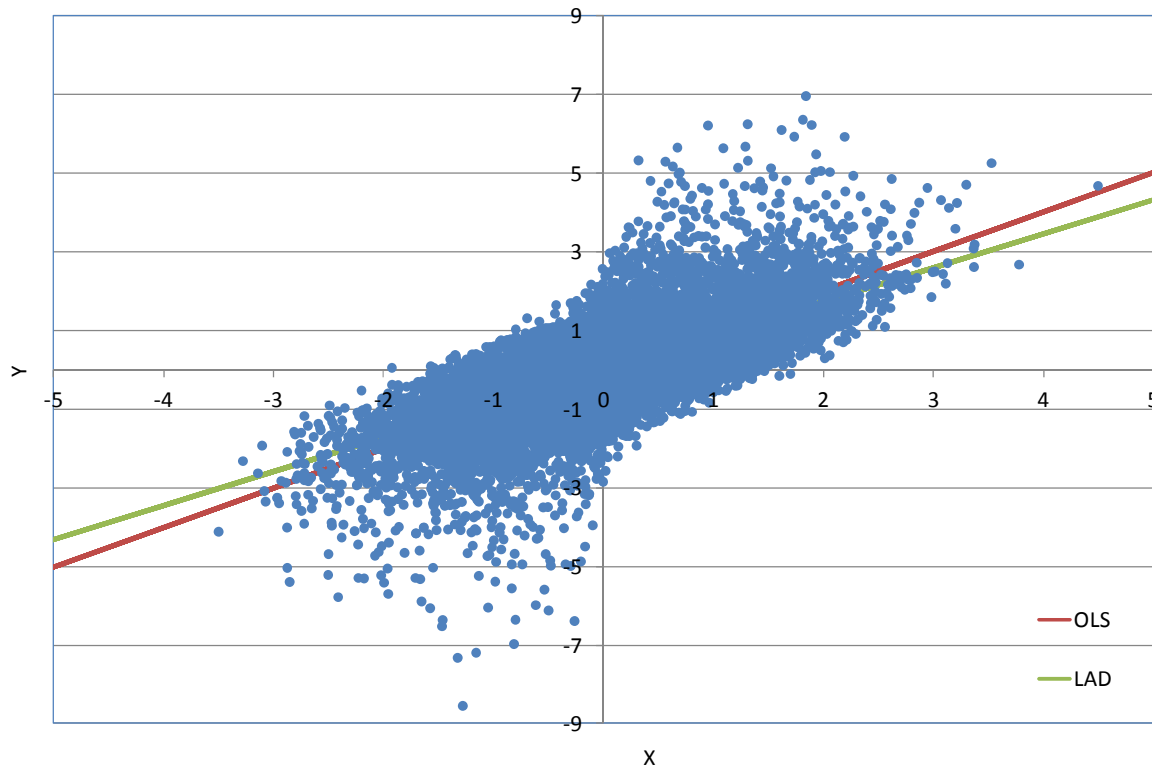
As the graph illustrates, when $\phi = 20$ the disturbance from the regression (5) is skewed to the left when $x_i < 0$ and skewed to the right when $x_i > 0$. Since LAD places less weight on extreme observations, the LAD estimate of the regression parameter β is biased downwards.

Table 2.1
Illustration of the bias that can be associated with LAD estimates of beta

ϕ	-20	-10	0	10	20
OLS	1.000 (0.101)	0.998 (0.102)	1.000 (0.102)	0.998 (0.101)	0.999 (0.101)
LAD	1.133 (0.111)	1.124 (0.111)	1.001 (0.118)	0.873 (0.109)	0.867 (0.110)

Notes: Mean beta estimates are outside of parentheses while the standard deviations of the estimates are inside parentheses. The simulations use 10,000 replications and for each replication 100 observations generated using (5) and (10).

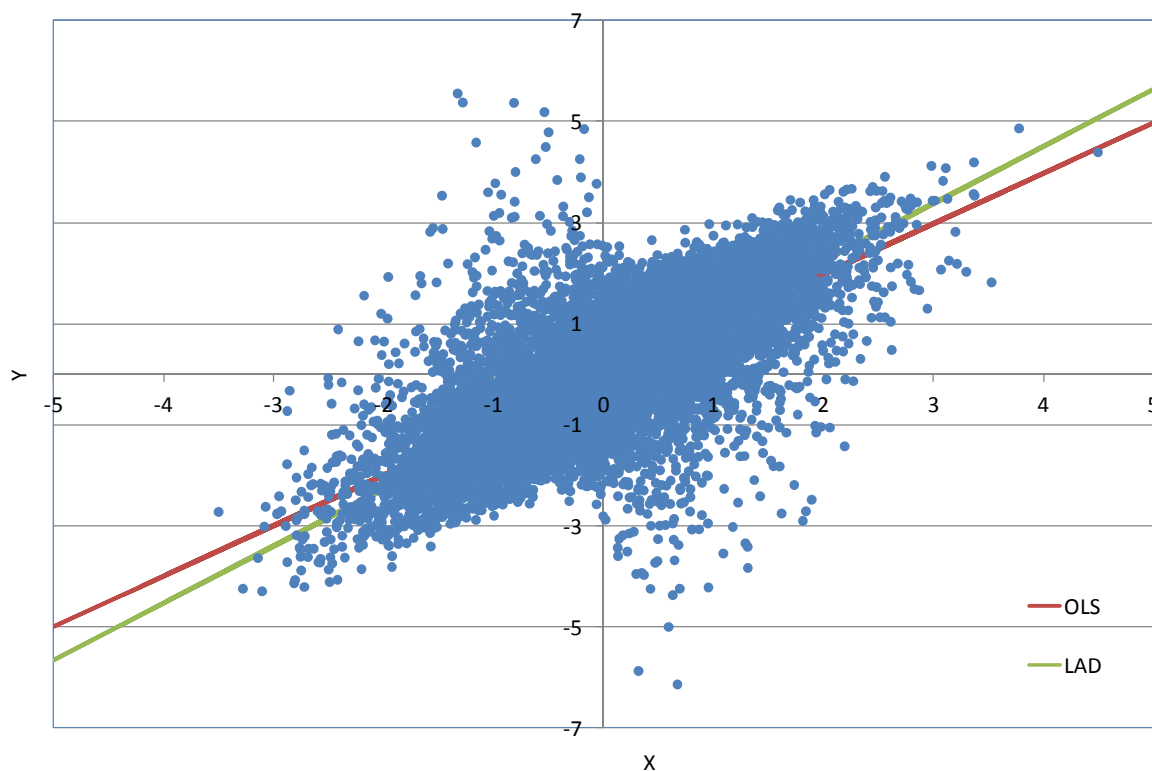
Figure 2.1
Illustration of the bias that can be associated with LAD estimates of beta: $\phi = 20$



Notes: OLS and LAD estimates are computed using 10,000 observations generated using (5) and (10) with $\phi = 20$.

Figure 2.2 provides a plot of y_t against x_t for a single sample of 10,000 observations when $\phi = -20$. When $\phi = -20$ the disturbance from the regression (5) is skewed to the right when $x_t < 0$ and skewed to the left when $x_t > 0$. Since LAD places less weight on extreme observations, the LAD estimate of the regression parameter β is biased upwards.

Figure 2.2
Illustration of the bias that can be associated with LAD estimates of beta: $\phi = -20$



Notes: OLS and LAD estimates are computed using 10,000 observations generated using (5) and (10) with $\phi = -20$.

3. Evidence

LAD estimators of the parameters of a regression will be maximum likelihood if the disturbance from the regression follows a Laplace distribution and so under this condition the estimators will be both consistent and asymptotically efficient.²⁷ The ERA in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013 provides the results of tests of the hypothesis that the disturbances from energy utility market model regressions follow a Laplace distribution.²⁸

OLS estimators of the parameters of a regression will be maximum likelihood if the disturbance from the regression is normally distributed. The ERA also provides the results of tests of the hypothesis that the disturbances from energy utility market model regressions are normally distributed.²⁹

In this section, like the ERA, we present the results of Gaussian and Laplace goodness-of-fit tests. Although our results differ somewhat from those that the ERA reports, we reach the same overall conclusion. The Gaussian goodness-of-fit tests provide evidence against the null hypothesis that the disturbances from energy utility market model regressions are normally distributed. The Laplace goodness-of-fit tests, in contrast, provide little evidence against the null hypothesis that the disturbances from energy utility market model regressions follow a Laplace distribution.

We also provide OLS and robust regression estimates of the equity beta of a regulated energy utility. We find that there can be marked differences between the OLS and robust regression estimates.

We, like the ERA, use goodness-of-fit tests suggested by Puig and Stephens (2000).³⁰ Puig and Stephens examine the power of their goodness-of-fit tests but only to detect one alternative distribution that is skewed – the skew extreme value distribution. Unfortunately, Puig and Stephens do not reveal the parameters of the skew extreme value distribution that they employ and so it is difficult to gauge the extent to which their results are relevant to the task of estimating the equity beta of a regulated energy utility. For this reason, in section 4, we conduct our own bootstrap simulations using a method that is identical to the method that the ERA employs.

3.1. Data

We extract data from Bloomberg for the six regulated energy utilities that the ERA employs: the APA Group, (APA), AusNet Services (AST), Duet Group (DUE), Envestra (ENV),

²⁷ Kennedy, P., *A guide to econometrics*, MIT Press, 1979, page 22.

Puig, P. and M.A. Stephens, *Tests of fit for the Laplace distribution with applications*, Technometrics, 2000, page 420.

²⁸ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

²⁹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

³⁰ Puig, P. and M.A. Stephens, *Tests of fit for the Laplace distribution with applications*, Technometrics, 2000, pages 417-424.

Hastings Diversified Utilities Fund (HDF) and Spark Infrastructure (SKI). We follow the ERA and use data from 19 April 2008 to 19 April 2013 although we note that HDF ceased trading on 23 November 2012. We extract from Bloomberg for each company, its fully adjusted price (PX_LAST), its unadjusted price (PX_LAST), its net debt (NET_DEBT) and the number of shares it has outstanding (EQY_SH_OUT). From the adjusted price, we compute the end-of-week to end-of-week return to each stock. From the unadjusted price and the number of shares outstanding, we compute the market capitalisation of each stock. From net debt, the unadjusted price and the number of shares outstanding, we compute the debt-to-value ratio of each stock; we compute the debt-to-value ratio as:

$$\frac{\text{NET_DEBT}}{\text{NET_DEBT} + \text{PX_LAST} \times \text{EQY_SH_OUT}}, \quad (11)$$

where here PX_LAST is the unadjusted price.

To compute the re-levered return to a stock, we multiply the un-re-levered return to the stock by:

$$\frac{1 - \bar{G}}{1 - 0.6}, \quad (12)$$

where $\bar{G}$ is the average debt-to-value ratio of the firm. For each firm, we compute $\bar{G}$ by averaging the debt-to-value ratio at the end of June and December of each year. We do so because net debt changes only once every six months.

Besides the re-levered returns to the six individual stocks, we also use the re-levered returns to two portfolios of the stocks: an equally weighted portfolio and a value-weighted portfolio. To compute the re-levered return to an equally weighted portfolio, each period we average the re-levered returns to the stocks among the six regulated energy utilities that trade. To compute the re-levered return to a value-weighted portfolio, we first compute the market-capitalisation weighted un-re-levered return to the portfolio. We then multiply this un-re-levered return by (12) with the average debt-to-value ratio of the firm replaced by the average debt-to-value ratio for the portfolio.³¹

³¹ We compute the debt-to-value ratio for the portfolio as the sum of net debt across the firms in the portfolio divided by the sum of net debt and the market value of equity across the firms in the portfolio.

3.2. Statistics

Puig and Stephens (2000) explain that in testing whether the disturbance from a regression is Laplace distributed that:³²

‘the statistic used should be one of W^2 , U^2 , or A^2 [the Cramér-von Mises statistic, Watson statistic or Anderson-Darling statistic] because only for these are the asymptotic distributions known.’

The ERA uses in addition to these three statistics, the Kolmogorov-Smirnov and Kuiper statistics.³³ We follow the advice of Puig and Stephens and so do not compute these statistics. Percentage points (critical values) for significance levels of five per cent taken from Puig and Stephens are 0.983 for the Anderson-Darling statistic, 0.144 for the Cramér-von Mises statistic and 0.084 for the Watson statistic.

Like the ERA, we use the Jarque-Bera statistic to test whether the disturbance from a regression is normally distributed.³⁴ Under the null hypothesis that the disturbance is normally distributed, the test statistic will be chi-square distributed with two degrees of freedom.

Like the ERA, we estimate the equity beta of a regulated utility using OLS, LAD, an MM estimator and the Theil-Sen methodology. We compute these estimates using SAS. In particular, we use PROC MODEL, the LAV routine in PROC IML, PROC ROBUSTREG and a routine in PROC IML to compute Theil-Sen estimates. Computing Theil-Sen estimates and their standard errors can be relatively time-consuming and so we compute the estimates but not their standard errors.³⁵

3.3. Results

3.3.1. Goodness-of-fit tests

Table 3.1 provides the results of Gaussian and Laplace goodness-of-fit tests. Like the ERA, we find that Jarques-Bera goodness-of-fit tests provide evidence against the null hypothesis that the disturbances from energy utility market model regressions are normally distributed. On the other hand, like the ERA, we find little evidence against the null hypothesis that the disturbances from energy utility market model regressions follow a Laplace distribution. The

³² Puig, P. and M.A. Stephens, *Tests of fit for the Laplace distribution with applications*, Technometrics, 2000, page 420.

³³ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

³⁴ For a discussion of how the Jarque-Bera test works, see:

Davidson, R. and J. G. MacKinnon, *Estimation and inference in econometrics*, Oxford University Press, Oxford, 1993, pages 568-569.

³⁵ Ohlson and Kim, for example, use simulations to construct the standard errors of the Theil-Sen estimates that they compute.

Ohlson, J.A. and S. Kim, *Linear valuation without OLS: The Theil-Sen estimation approach*, NYU working paper, 2014, page 23.

only stock or portfolio for which we find evidence against this null at the five per cent level is for HDF.

Table 3.1
Laplace tests of fit computed using weekly data from 19 April 2008 to 19 April 2013

	Jarque-Bera	Anderson-Darling	Cramér von Mises	Watson
Panel A: Individual stocks				
APA	47.344	0.667	0.117	0.068
AST	49.302	0.331	0.050	0.047
DUE	1,545.855	0.395	0.048	0.047
ENV	427.442	0.492	0.058	0.032
HDF	4,660.089	1.868	0.270	0.258
SKI	429.984	0.356	0.044	0.038
Panel B: Portfolios				
EW	1,201.107	0.568	0.080	0.049
VW	159.653	0.440	0.036	0.036

Notes: Jarque-Bera (Anderson-Darling, Cramér von Mises and Watson) statistics that indicate that one can reject at the five percent level the null hypothesis that the disturbance from a market model regression is normally (Laplace) distributed are in bold. Inference for the Jarque-Bera tests is based on a comparison of the statistics in the table with the percentage points (critical values) of a chi-square with two degrees of freedom. Inference for the Anderson-Darling, Cramér von Mises and Watson tests is based on a comparison of the statistics in the table with the percentage points (critical values) provided by Puig and Stephens (2000) in their Tables 1 to 3.

Puig, P. and M.A. Stephens, Tests of fit for the Laplace distribution with applications, Technometrics, 2000, pages 417-424.

The goodness-of-fit tests that appear in Table 3.1 test the null hypotheses that the disturbance from a market model regression is normally distributed or is Laplace distributed. We note that the normal distribution and the Laplace distribution are special cases of the distribution:³⁶

$$f(u) = k(\sigma, \theta) \exp\left(-\left|\frac{u}{\sigma}\right|^\theta\right), \quad k(\sigma, \theta) = \left[2\sigma\Gamma\left(1+\frac{1}{\theta}\right)\right]^{-1}, \quad \sigma > 0, \theta > 0 \quad (13)$$

³⁶ See Maddala (1977) and the references therein.

Maddala, G.S., *Econometrics*, McGraw-Hill, New York, 1977, pages 308-314.

When $\theta = 1$, (13) describes a Laplace distribution. When $\theta = 2$, (13) describes a normal distribution. It follows that one can use the method of maximum likelihood to estimate the parameter θ . In other words one can let the data decide which of the two distributions best describe the data. When we use this method we find, consistent with the goodness-of-fit results that we report in Table 3.1, that the disturbances from an energy utility market model regression are typically better described by a Laplace distribution than by a normal distribution.

It is important to note, however, that while it may be true that the disturbances from an energy utility market model regression are typically better described by a Laplace distribution than by a normal distribution, neither distribution may correctly describe how the disturbances are distributed. The distributions described by (13), for example, are symmetric and it may be that the data would be better described by a distribution that is not symmetric.³⁷

It is also important to note that while LAD estimators will no longer be maximum likelihood when the Laplace assumption is violated and so can lose some of their desirable properties—for example, the property of consistency – OLS estimators, while no longer maximum likelihood where the data are not normally distributed, can still retain many of their desirable properties. For example, as Wooldridge (2013) makes clear, even when the disturbances from a regression are not normally distributed, OLS estimators can still be Best Linear Unbiased.³⁸ The ERA, in contrast, asserts, incorrectly, that a necessary condition for OLS estimators to be Best Linear Unbiased is that the disturbance from a regression be normally distributed.³⁹

3.3.2. Beta estimates

We also provide OLS and robust regression estimates of the equity beta of a regulated energy utility. Table 3.2 provides estimates of the equity beta of each firm and of the two portfolios using OLS and the three robust regression techniques. The table shows that there are differences in the estimates both across stocks but also across estimation methods. The OLS estimate of the equity beta of AST is just 0.260 while the OLS estimate of the equity beta of HDF is 1.134. But while the OLS estimate of the equity beta of HDF is 1.134, the MM estimate of the equity beta, computed using exactly the same data, is 0.914.

To examine what is causing the differences that we observe between OLS and LAD estimates, we plot the re-levered returns to AST, HDF, SKI and the value-weighted portfolio against the return to the market. These are the three stocks and one portfolio for which there are the largest differences between the OLS estimates of beta and the robust regression estimates.

³⁷ The Laplace distribution is also known – reflecting its symmetry – as the double exponential distribution. See Maddala (1977).

Maddala, G.S., *Econometrics*, McGraw-Hill, New York, 1977, pages 312.

³⁸ Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 102.

³⁹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, page 166.

Figure 3.1 plots the return to AST against the return to the market. It can be seen that this figure bears some resemblance to Figure 3.2. The distribution of market model residuals appears to be skewed to the right when the market return is below its mean and skewed to the left when the market return is above its mean. As a result, the OLS estimate of beta is below the LAD estimate and the other robust regression estimates. Figures 3.2 and 3.3 plot the returns to HDF and SKI against the return to the market. These figures bear some resemblance to Figure 2.1. The distribution of market model residuals in both figures appears to be skewed to the left when the market return is below its mean and skewed to the right when the market return is above its mean. As a result, the OLS estimates of beta lie above the corresponding LAD estimates and the other robust regression estimates.

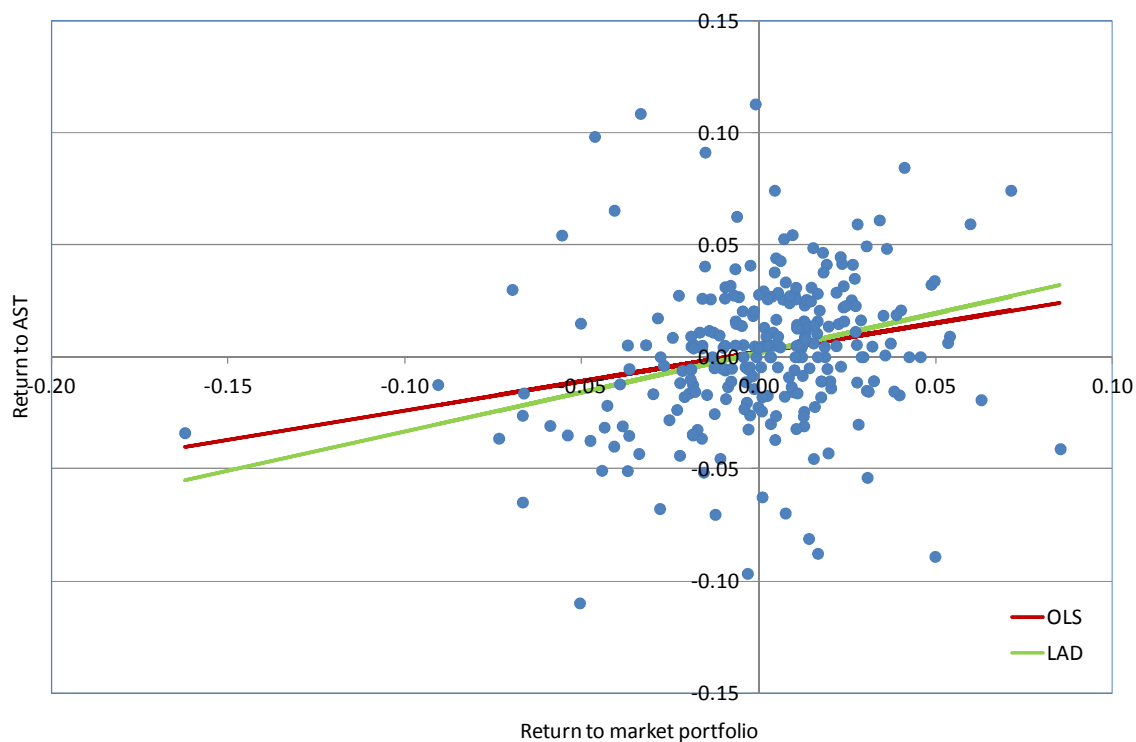
Figure 3.4 plots the return to the value-weighted portfolio against the return to the market. This figure does not closely resemble either Figure 2.1 or Figure 2.2. Nevertheless, the OLS estimate of beta is below the LAD estimate and the other robust regression estimates.

Table 3.2
Estimates of the equity beta of a firm computed using weekly data
from 19 April 2008 to 19 April 2013

	OLS		LAD		MM		T-S
	Estimate	Asymptotic std. error	Estimate	Asymptotic std. error	Estimate	Asymptotic std. error	Estimate
Panel A: Individual stocks							
APA	0.567	0.076	0.543	0.071	0.591	0.067	0.535
AST	0.260	0.071	0.352	0.077	0.380	0.062	0.339
DUE	0.263	0.054	0.219	0.044	0.233	0.041	0.262
ENV	0.400	0.067	0.399	0.052	0.410	0.052	0.404
HDF	1.134	0.217	1.001	0.113	0.914	0.111	0.934
SKI	0.527	0.122	0.402	0.104	0.453	0.106	0.391
Panel B: Portfolios							
EW	0.522	0.063	0.531	0.052	0.511	0.047	0.497
VW	0.387	0.049	0.460	0.047	0.421	0.041	0.409

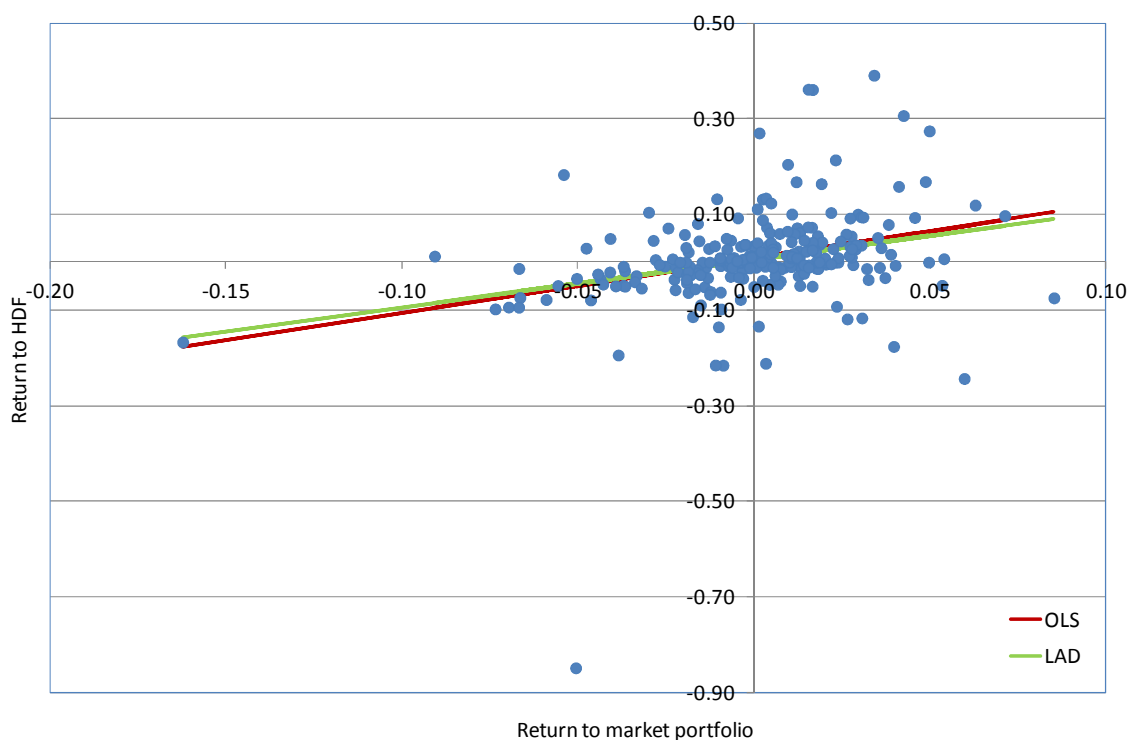
Notes: T-S estimates are Theil-Sen estimates.

Figure 3.1
Returns to AST against returns to market portfolio



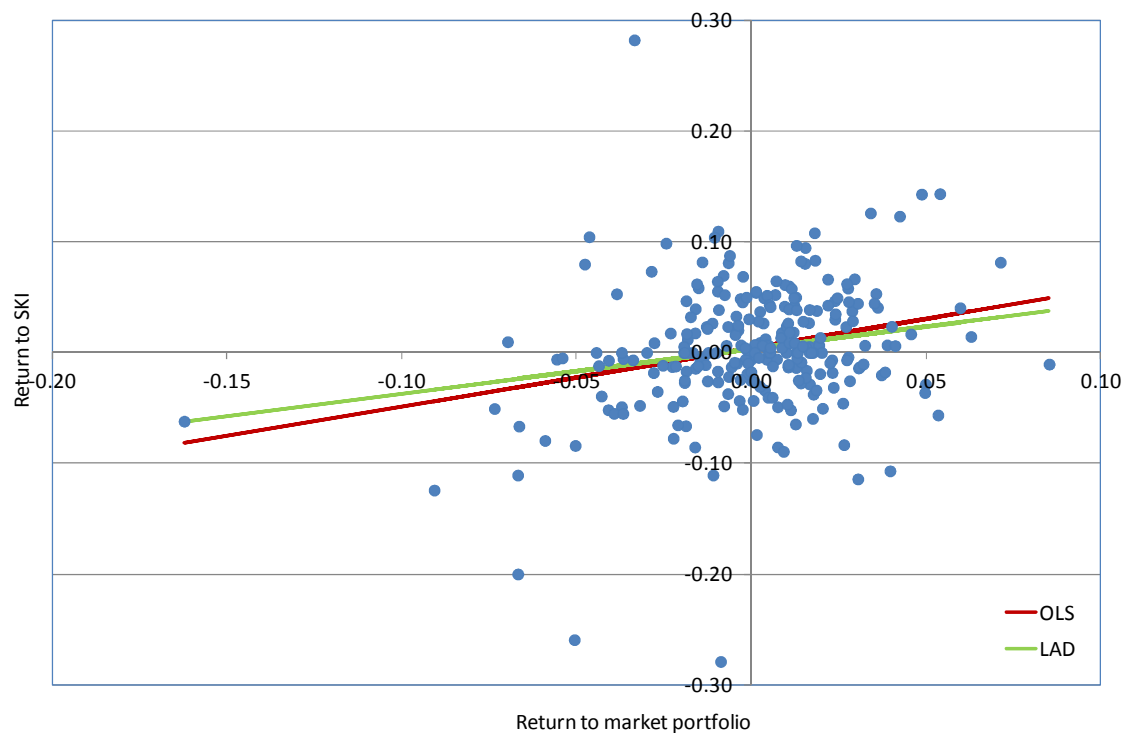
Notes: The plot uses weekly re-levered returns from 19 April 2008 to 19 April 2013.

Figure 3.2
Returns to HDF against returns to market portfolio



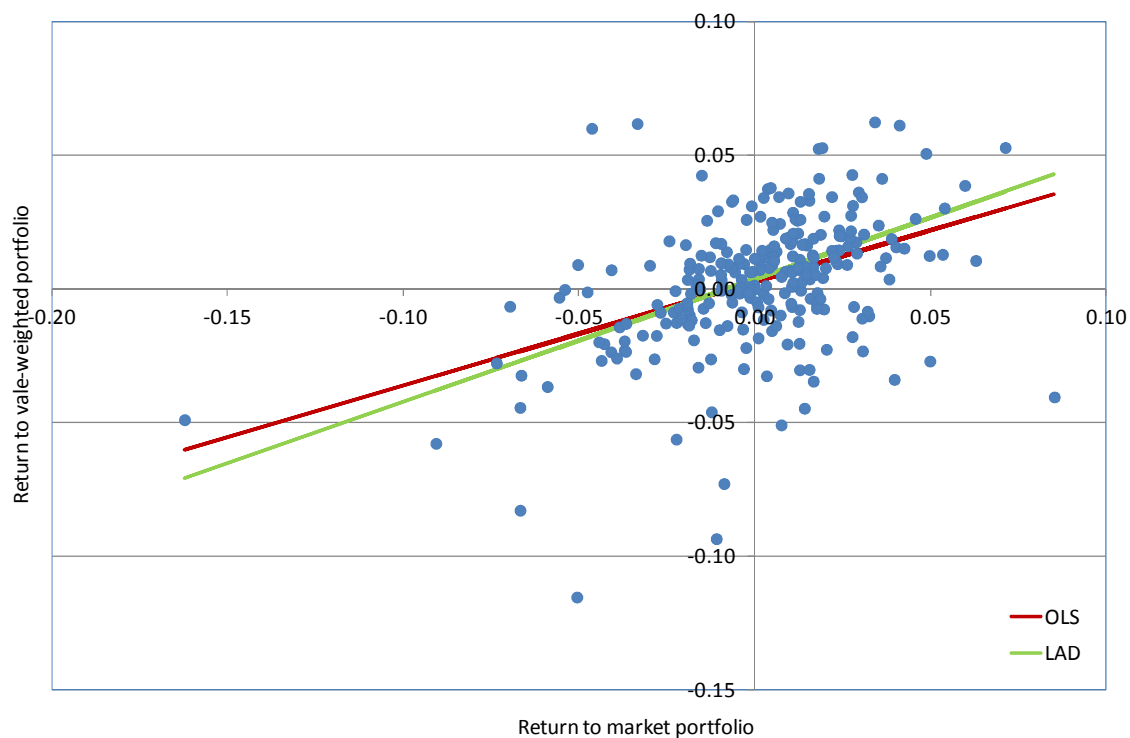
Notes: The plot uses weekly re-levered returns from 19 April 2008 to 19 April 2013.

Figure 3.3
Returns to SKI against returns to market portfolio



Notes: The plot uses weekly re-levered returns from 19 April 2008 to 19 April 2013.

Figure 3.4
Returns to value-weighted portfolio against returns to market portfolio



Notes: The plot uses weekly re-levered returns from 19 April 2008 to 19 April 2013.

4. Simulations

The ERA in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013 provides the results of bootstrap simulations that examine the behaviour of robust regression estimates.⁴⁰ Surprisingly, the regulator does not report the means of the simulated distributions. So one cannot infer from the regulator's results whether the robust regression estimates are biased. Also, the regulator does not take the opportunity of assessing the ability of the Puig-Stephens goodness-of-fit tests to detect departures from the Laplace null hypothesis. Nor does the regulator take the opportunity of assessing a strategy, that it appears to recommend, of using a LAD estimator unless a goodness-of-fit test indicates that one should do otherwise.

In this section, we conduct bootstrap simulations like those that the ERA uses. We find that:

- the three robust regression techniques typically provide biased estimates of beta whereas the OLS estimates exhibit no significant bias;
- the three robust regression techniques typically provide estimates of beta that are more precise;
- the tests that Puig and Stephens advocate one use can fail to detect departures from the Laplace null hypothesis; and
- a strategy of using a Laplace goodness-of-fit test to determine whether to use OLS or LAD does not perform well – for four of the six stocks and for one of the two portfolios OLS estimates display both less bias and a lower variability than estimates that use the strategy.

4.1. Methodology

For each stock and portfolio we form a $T \times 2$ matrix, where T denotes the number of weeks of data that are available over the period 19 April 2008 to 19 April 2013. In the first column we place the re-levered weekly returns to the stock or portfolio while in the second column we place the corresponding weekly returns to the market. We form samples of 260 weeks data and samples of 100,000 weeks of data by sampling from each row with replacement. The simulations that we conduct are based on 10,000 replications when we use 260 weeks of data and 1,000 replications when we use 100,000 weeks of data.

4.2. Results

Table 4.1 provides the results of simulations in which each replication uses a sample of 260 weeks of data. A comparison of Table 5.1 with Table 4.2 shows that, not surprisingly, OLS estimators are unbiased. Note that since the bootstrap simulations use data sampled from the series that Table 4.2 employs, then by construction the OLS estimates that appear in Table 4.2 are, for purposes of the simulations, the true parameters. On the other hand, many of the robust regression estimators are biased. Table 4.1 indicates, for example, that the

⁴⁰ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 163-180.

Theil-Sen estimator is typically biased.⁴¹ The robust regression estimators are, however, typically more precise than their OLS counterparts.

Table 4.1
Simulation evidence for beta estimates that use 260 weeks of data

	OLS		LAD		MM		T-S
	Estimate	Asymptotic std. error	Estimate	Asymptotic std. error	Estimate	Asymptotic std. error	Estimate
Panel A: Individual stocks							
APA	0.565 (0.090)	0.076 (0.007)	0.544 (0.089)	0.076 (0.014)	0.584 (0.077)	0.067 (0.007)	0.533 (0.079)
AST	0.262 (0.084)	0.072 (0.007)	0.344 (0.108)	0.071 (0.014)	0.386 (0.084)	0.063 (0.006)	0.341 (0.082)
DUE	0.267 (0.074)	0.054 (0.007)	0.221 (0.068)	0.045 (0.009)	0.237 (0.073)	0.041 (0.004)	0.262 (0.057)
ENV	0.402 (0.080)	0.067 (0.008)	0.412 (0.060)	0.053 (0.011)	0.412 (0.061)	0.052 (0.006)	0.405 (0.066)
HDF	1.141 (0.295)	0.206 (0.033)	0.973 (0.150)	0.110 (0.022)	0.904 (0.134)	0.108 (0.013)	0.938 (0.146)
SKI	0.524 (0.154)	0.122 (0.014)	0.381 (0.151)	0.104 (0.024)	0.456 (0.132)	0.107 (0.012)	0.393 (0.137)
Panel B: Portfolios							
EW	0.523 (0.085)	0.063 (0.008)	0.527 (0.056)	0.053 (0.010)	0.511 (0.0560)	0.048 (0.006)	0.496 (0.057)
VW	0.388 (0.065)	0.050 (0.005)	0.444 (0.061)	0.047 (0.009)	0.424 (0.048)	0.041 (0.004)	0.411 (0.052)

Notes: Mean beta estimates and mean estimated asymptotic standard errors are outside of parentheses while the standard deviations of the estimates and standard errors are inside parentheses. The simulations use 10,000 replications.

⁴¹ The ERA states, on the other hand, that:

‘The Theil-Sen estimator is an unbiased ... estimator of the true parameter to be estimated.’

This statement, as Wang and Yu (2005) show, is incorrect. They note that the estimator:

‘is biased in general.’

Wang, X. and Q. Yu, *Unbiasedness of the Theil-Sen estimator*. Journal of Nonparametric Statistics, 2005, page 687.

Table 4.2 provides evidence on the power of Laplace goodness-of-fit tests. The table indicates that the tests are able to successfully detect departures from the null for HDF but are less successful at detecting departures for the other stocks and the two portfolios – even though the evidence in Table 4.1 indicates that there are costs – in the form of bias – associated with the departures.

Table 4.2
Simulation evidence for goodness-of-fit tests that use 260 weeks of data

Test size	Probability that statistic will reject null							
	Jarques-Bera		Anderson-Darling		Cramér von Mises		Watson	
	5%	1%	5%	1%	5%	1%	5%	1%
Panel A: Individual stocks								
APA	0.983	0.955	0.276	0.065	0.288	0.082	0.529	0.236
AST	0.999	0.995	0.239	0.063	0.242	0.072	0.404	0.169
DUE	1.000	1.000	0.356	0.123	0.285	0.092	0.494	0.244
ENV	1.000	0.999	0.361	0.117	0.247	0.074	0.335	0.108
HDF	1.000	1.000	0.982	0.900	0.967	0.856	0.994	0.975
SKI	0.977	0.968	0.381	0.142	0.345	0.138	0.503	0.229
Panel B: Portfolios								
PRT	0.999	0.998	0.422	0.174	0.361	0.153	0.508	0.244
VRT	1.000	0.999	0.458	0.158	0.331	0.127	0.530	0.250

Notes: The simulations use 10,000 replications. Inference is based on a comparison of the statistics with the asymptotic percentage points (critical values) provided by Puig and Stephens (2000) in their Tables 1 to 3.

Puig, P. and M.A. Stephens, Tests of fit for the Laplace distribution with applications, Technometrics, 2000, pages 417-424.

Table 4.3 examines the properties of the strategy that the ERA appears to endorse of using a LAD estimator unless a goodness-of-fit test indicates that one should do otherwise. The table indicates that the strategy does not perform well – for four of the six stocks and for one of the two portfolios OLS estimates display both less bias and a lower variability than estimates that use the strategy. Where OLS estimates dominate those produced by the ERA strategy, we highlight the results in bold.

Table 4.3
Simulation evidence on the ERA screening strategy that use 260 weeks of data

Method	APA	AST	DUE	ENV	HDF	SKI	EW	VW
AND	0.550 (0.092)	0.326 (0.107)	0.240 (0.076)	0.405 (0.069)	1.142 (0.293)	0.432 (0.163)	0.533 (0.073)	0.419 (0.069)
CRA	0.549 (0.091)	0.327 (0.108)	0.236 (0.075)	0.408 (0.067)	1.142 (0.293)	0.427 (0.159)	0.531 (0.070)	0.426 (0.069)
WAT	0.553 (0.091)	0.313 (0.104)	0.246 (0.076)	0.408 (0.069)	1.141 (0.294)	0.450 (0.159)	0.532 (0.075)	0.414 (0.070)
OLS	0.565 (0.090)	0.262 (0.084)	0.267 (0.074)	0.402 (0.080)	1.141 (0.295)	0.524 (0.154)	0.523 (0.085)	0.388 (0.065)
LAD	0.544 (0.089)	0.344 (0.108)	0.221 (0.068)	0.412 (0.060)	0.973 (0.150)	0.381 (0.151)	0.527 (0.056)	0.444 (0.061)

Notes: The simulations use 10,000 replications. Inference is based on a comparison of the statistics with the asymptotic percentage points (critical values) provided by Puig and Stephens (2000) in their Tables 1 to 3. The AND method uses the LAD estimator unless the Anderson-Darling test rejects the null that the disturbance from a market model regression is Laplace distributed – in which case the OLS estimator is employed. The CRA method uses the LAD estimator unless the Cramér-von Mises test rejects the null that the disturbance from a market model regression is Laplace distributed – in which case the OLS estimator is employed. The WAT method uses the LAD estimator unless the Watson test rejects the null that the disturbance from a market model regression is Laplace distributed – in which case the OLS estimator is employed. Companies for which the OLS method dominates (lower bias and lower standard deviation) the ERA strategy are highlighted in bold.

Puig, P. and M.A. Stephens, Tests of fit for the Laplace distribution with applications, Technometrics, 2000, pages 417-424.

Finally, Table 4.4 shows that the bias associated with the robust regression estimators is not a small-sample phenomenon. The results in the table are generated by conducting simulations that use 1,000 replications where each replication employs a sample of 100,000 weeks. The large-sample mean beta estimates are similar to their small-sample counterparts in Table 4.1.

Table 4.4
Simulation evidence for beta estimates that use 100,000 weeks of data

	OLS		LAD		MM	
	Estimate	Asymptotic std. error	Estimate	Asymptotic std. error	Estimate	Asymptotic std. error
Panel A: Individual stocks						
APA	0.568 (0.004)	0.004 (0.000)	0.538 (0.023)	0.013 (0.005)	0.591 (0.004)	0.003 (0.000)
AST	0.260 (0.004)	0.004 (0.000)	0.357 (0.008)	0.014 (0.005)	0.381 (0.004)	0.003 (0.000)
DUE	0.263 (0.004)	0.003 (0.000)	0.223 (0.005)	0.006 (0.002)	0.233 (0.004)	0.002 (0.000)
ENV	0.400 (0.004)	0.003 (0.000)	0.399 (0.000)	0.001 (0.001)	0.410 (0.003)	0.003 (0.000)
HDF	1.134 (0.015)	0.011 (0.000)	1.014 (0.014)	0.007 (0.002)	0.915 (0.007)	0.005 (0.000)
SKI	0.527 (0.007)	0.006 (0.000)	0.380 (0.029)	0.014 (0.004)	0.453 (0.006)	0.005 (0.000)
Panel B: Portfolios						
EW	0.522 (0.004)	0.003 (0.000)	0.529 (0.007)	0.005 (0.002)	0.511 (0.002)	0.002 (0.000)
VW	0.387 (0.003)	0.003 (0.000)	0.454 (0.012)	0.006 (0.003)	0.421 (0.002)	0.002 (0.000)

Notes: Mean beta estimates and mean estimated asymptotic standard errors are outside of parentheses while the standard deviations of the estimates and standard errors are inside parentheses. The simulations use 1,000 replications.

5. Literature Review

If robust regression techniques offer important advantages over ordinary least squares, there are few disadvantages to using the techniques and the market for academic research is efficient, then one should expect to find evidence of the frequent use of these techniques in published work.

In this section, we conduct keyword searches of the four major finance journals as a way of discovering how frequently robust regression techniques are used in high quality research relative to ordinary least squares.

The four journals that we select are the Journal of Finance, the Journal of Financial Economics, the Journal of Financial and Quantitative Analysis and the Review of Financial Studies. These are the four finance journals included in the list of 45 journals used by the Financial Times in compiling its business school research rankings.⁴² They are also the four journals which a recent study of finance journal rankings that Currie and Pandher (2010) conduct rate most highly in terms of their quality.⁴³

We find that in these four journals there are relatively few references to robust regression techniques. We search for references to the phrase ‘ordinary least squares’ and references to nine phrases that concern three different robust regression techniques. Across the Journal of Finance, the Journal of Financial Economics and the Review of Financial Studies – the journals that Currie and Pandher rate most highly in terms of quality – we find 1,449 references to the phrase ‘ordinary least squares’ and 75 references to the nine phrases that concern robust regression techniques. In the Journal of Financial and Quantitative Analysis we find surprisingly few references to any of the 10 phrases. We find 27 references to the phrase ‘ordinary least squares’ and 8 references to the nine phrases that concern robust regression techniques.

Thus the evidence that we provide strongly suggests that robust regression techniques are used infrequently in high quality finance research.

5.1. Journal of Finance

We begin by conducting keyword searches of the Journal of Finance. We search for the phrase ‘ordinary least squares’ and nine phrases that concern three different robust regression techniques. Five of these nine phrases concern LAD. These five phrases are:

- least absolute deviation;
- least absolute error;
- least absolute residual;
- minimum absolute deviation; and

⁴² <http://www.ft.com/intl/cms/s/2/3405a512-5cbb-11e1-8f1f-00144feabdc0.html#axzz3LBbELIWS>

⁴³ Currie, R.R. and G. S. Pandher, *Finance journal rankings and tiers: An active scholar assessment methodology*, Journal of Banking and Finance, 2011, pages 7-20.

- minimizing the sum of absolute errors.

Two of the nine phrases concern MM estimation. These two phrases are:

- MM estimate; and
- MM estimator.

The last two phrases are:

- robust regression; and
- Theil-Sen.

We also searched for the following minor variations on the nine phrases:

- least absolute deviations;
- least absolute errors;
- least absolute residuals;
- minimum absolute deviations;
- MM estimates; and
- MM estimators.

We treat multiple references to the same phrase or two very closely related phrases – for example, to ‘least absolute deviation’ and to ‘least absolute deviations’ – as a single reference. Also, we combine references to two very closely related phrases – eliminating double counting – and report the number we find under a single heading. Thus we do not report references to ‘least absolute deviation’ and to ‘least absolute deviations’ separately. We report them together under the heading ‘least absolute deviation’.

We do not search for references to acronyms. This is for two reasons. First, by searching for the 10 phrases together with a number of acronyms we run the risk of double counting.⁴⁴ Second, and more importantly, a reference to an acronym may have nothing to do with the choices authors make about how to estimate parameters. As an example, the first appearance of the expression ‘LAD’ in the *Journal of Finance* occurs in 1943 as part of the word ‘maladjustments’.⁴⁵

⁴⁴ By searching for a reference to the phrase ‘robust regression’ as well as for references to the use of particular robust regression techniques, we already run this risk. So, even without searching for acronyms, our results will be biased towards finding relatively more references to robust regression techniques than to ordinary least squares.

⁴⁵ Graham (1949) states on page 17 of his paper:

‘Like all such attempts this policy merely cumulated maladjustments until the British lost all their gold and the £ fell with a crash in 1931.’

Graham, F.D., *Exchange rates: Bound or free*, *Journal of Finance*, 1949, pages 13-27.

Table 5.1 provides the results of our search for the 10 phrases for the Journal of Finance. The first issue of the Journal of Finance appeared in 1946. We find references to the phrases from 1967 to 2014. In particular, we find 582 references to the phrase ‘ordinary least squares’ and 33 references to phrases concerning robust regression techniques. Thus the evidence that we provide from a search of the Journal of Finance strongly suggests that robust regression techniques are used infrequently in research published in the Journal.

Table 5.1
Keyword search results for the Journal of Finance

Phrase	Number of cites	First	Last
Ordinary least squares	582	1967	2014
Least absolute deviation	10	1996	2012
Least absolute error	0		
Least absolute residual	1	1972	1972
Minimum absolute deviation	3	1986	2004
Minimizing the sum of absolute errors	0		
MM estimate	2	1980	1980
MM estimator	0		
Robust regression	17	1986	2013
Theil-Sen	0		

Source: [http://onlinelibrary.wiley.com/journal/10.1111/\(ISSN\)1540-6261](http://onlinelibrary.wiley.com/journal/10.1111/(ISSN)1540-6261)

Note: Multiple multiple references to the same phrase or two very closely related phrases are treated as a single reference.

5.2. Journal of Financial Economics

We next conduct keyword searches of the Journal of Financial Economics. We search for references to the same 10 phrases and provide the results of doing so in Table 5.2.

We find references to the phrases from 1974 – the first year in which the Journal was published – to 2014. In particular, we find 699 references to the phrase ‘ordinary least squares’ and 29 references to phrases concerning robust regression techniques. Thus the evidence that we provide from a search of the Journal of Financial Economics is very similar to the evidence that we provide from a search of the Journal of Finance. That is to say, the evidence that we provide from a search of the Journal of Financial Economics strongly suggests that robust regression techniques are used infrequently in research published in the Journal.

Table 5.2
Keyword search results for the Journal of Financial Economics

Phrase	Number of cites	First	Last
Ordinary least squares	699	1974	2014
Least absolute deviation	10	2006	2014
Least absolute error	1	1980	1980
Least absolute residual	0		
Minimum absolute deviation	0		
Minimizing the sum of absolute errors	1	2006	2006
MM estimate	0		
MM estimator	0		
Robust regression	17	1983	2011
Theil-Sen	0		

Source: <http://www.sciencedirect.com/science/journal/0304405X>

Note: Multiple multiple references to the same phrase or two very closely related phrases are treated as a single reference.

5.3. Journal of Financial and Quantitative Analysis

A search of the Journal of Financial and Quantitative Analysis (JFQA) reveals a surprisingly small number of references to the 10 phrases.

As Table 5.3 indicates, we find references to the phrases from 1967 – one year after the Journal was first published – to 2014. We find, however, only 27 references to the phrase ‘ordinary least squares’ and just eight references to phrases that concern robust regression techniques. The very small number of references that we find to the 10 phrases for the JFQA relative to the number of references we find for the Journal of Finance and the Journal of Financial Economics suggests that the search engine that the JFQA uses may not successfully unearth all references that exist. Thus it would make sense to treat the results of our search of the JFQA with caution.

Table 5.3
Keyword search results for the Journal of Financial and Quantitative Analysis

Phrase	Number of cites	First	Last
Ordinary least squares	27	1967	2014
Least absolute deviation	1	2014	2014
Least absolute error	2	1978	1978
Least absolute residual	0		
Minimum absolute deviation	0		
Minimizing the sum of absolute errors	1	1978	1978
MM estimate	0		
MM estimator	0		
Robust regression	4	1975	2005
Theil-Sen	0		

Source: <http://journals.cambridge.org/action/displayJournal?jid=JFQ>

Note: Multiple multiple references to the same phrase or two very closely related phrases are treated as a single reference.

5.4. Review of Financial Studies

Finally, we conduct keyword searches of the Review of Financial Studies, for references, again, to the same 10 phrases. We provide the results of these searches in Table 5.4.

We find references to the phrases from 1992 – four years after the Journal was first published – to 2014. In particular, we find 168 references to the phrase ‘ordinary least squares’ and 13 references to phrases concerning robust regression techniques. So the evidence that we provide from a search of the Review of Financial Studies is very similar to the evidence that we provide from a search of the Journal of Finance and from a search of the Journal of Financial Economics. That is to say, the evidence that we provide from a search of the Review of Financial Studies suggests that robust regression techniques are used infrequently in research published in the Journal.

5.5. Discussion

Our results strongly suggest that robust regression techniques are rarely used in research in finance. There are a number of reasons why this may be so and here we discuss what these are.

Table 5.4
Keyword search results for the Review of Financial Studies

Phrase	Number of cites	First	Last
Ordinary least squares	168	1992	2014
Least absolute deviation	4	2001	2012
Least absolute error	0		
Least absolute residual	0		
Minimum absolute deviation	1	2003	2003
Minimizing the sum of absolute errors	0		
MM estimate	0		
MM estimator	1	2010	2010
Robust regression	7	1998	2012
Theil-Sen	0		

Source: <http://rfs.oxfordjournals.org/search>

Note: Multiple multiple references to the same phrase or two very closely related phrases are treated as a single reference.

The benefits of using robust regression techniques instead of ordinary least squares are that parameter estimates generated by robust regression techniques can be more precise than estimates generated by ordinary least squares when the data are fat-tailed.⁴⁶ The costs of using robust regression techniques instead of ordinary least squares are that parameter estimates generated by robust regression techniques can be biased when estimates generated by ordinary least squares are not. So whether one will choose to use robust regression

⁴⁶ The precision of a random variable is the reciprocal of its variance. This definition, standard in the statistics literature, differs from the Oxford Dictionary definition of precision which is:

‘accuracy or exactness.’

In statistics a precise estimator can be exact but inaccurate. As Davidson and MacKinnon note, however,

‘it is sometimes more intuitive to think in terms of precision than in terms of variance.’

We agree and so use the terms precise and precision to render our discussion easier to follow.

Davidson, R. and J. G. MacKinnon, *Estimation and inference in econometrics*, Oxford University Press, Oxford, 1993, page 144.

Fowler, F.G. and H.W. Fowler, *Pocket Oxford Dictionary*, Oxford University Press, Oxford, 1966, page 623.

techniques instead of ordinary least squares to estimate a set of parameters can hinge on the importance that one attaches to bias relative to the importance that one attaches to precision.

Academic research often uses large samples and these samples can be diversified across industries and even, sometimes, across countries. Academics like using large samples because, all else constant, the larger a sample the more precise estimates produced from the sample. The use of large samples drawn from different industries and countries may therefore lead researchers to worry more about bias than precision.

In addition, academic research often examines events occurring at different points in time – in the language of the finance literature, events that are not clustered.⁴⁷ Academics like examining events that occur at different points in time because it can enable them to diversify away the impact of confounding factors and so better reveal the impact of the events. The use of samples drawn from different time periods may also lead researchers to worry more about bias than precision.

Regardless of why the academic literature makes the choices that it does, however, our evidence indicates that robust estimators are very rarely used in research in finance. This conclusion matches the conclusion that Ohlson and Kim (2014) draw about the use of robust estimators in research in accounting.⁴⁸ They state about the Theil-Sen (TS) method that:⁴⁹

‘There is an extensive statistics literature on robust estimators, TS being only one out of a vast set. These methodologies have not left much of a trace in the published accounting literature; the main alternative to OLS that has been tried out is LAD.’

Ohlson and Kim conclude that the Theil-Sen method provides an attractive alternative to the use of ordinary least squares although they caution that:

‘more experience with the TS method may expose all sorts of potential disadvantages.’

Tellingly, however, Ohlson and Kim do not consider the issue of bias –an important, as we show, potential disadvantage.⁵⁰

⁴⁷ See

Campbell, J.Y., A.W. Lo and A.C. MacKinlay, *The econometrics of financial markets*, Princeton University Press, 1997, Chapter 4.

⁴⁸ Ohlson, J.A. and S. Kim, *Linear valuation without OLS: The Theil-Sen estimation approach*, NYU working paper, 2014.

⁴⁹ Ohlson, J.A. and S. Kim, *Linear valuation without OLS: The Theil-Sen estimation approach*, NYU working paper, 2014, page 23.

⁵⁰ A search of their paper finds three references to the word ‘bias’. Two of these references are to Wang and Yu (2005) who provide conditions under which the Theil-Sen estimator will be unbiased while the third presumes that these conditions will be met.

Wang, X. and Q. Yu, *Unbiasedness of the Theil-Sen estimator*, Journal of Nonparametric Statistics, 2005, pages 685-695.

6. Conclusions

This report has been prepared for DBP by NERA Economic Consulting (NERA). The Economic Regulation Authority (ERA), in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, advocates the use of robust regression techniques to estimate the equity beta of a regulated energy utility.⁵¹ DBP has asked NERA to assess the costs and benefits of using robust regression techniques to estimate the equity beta of a regulated energy utility.

It is well known that a benefit to using robust regression estimates is that they are less sensitive to extreme observations and so typically less variable than their ordinary least squares (OLS) counterparts. It is less well known that a cost associated with the use of robust regression estimates is that the estimates can be biased. DBP has asked NERA:

- to show how a bias associated with robust regression estimates can arise.

The ERA, in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, suggests that one choose between least absolute deviations (LAD) and OLS estimators on the basis of goodness-of-fit tests.⁵² The ERA in the appendices also examines the behaviour of OLS and robust regression estimators using bootstrap simulations.⁵³ DBP has asked NERA:

- to use bootstrap simulations like those that the ERA employs to examine the behaviour of estimators constructed using a goodness-of-fit screening strategy of the kind that the ERA suggests one adopt.

If the benefits of using robust regression techniques outweigh the costs and the market for academic research is efficient, then one should expect to find evidence of the frequent use of the techniques in published work. Thus DBP has also asked NERA:

- to review the finance literature to determine the extent to which robust estimation techniques are used in research.

Bias

Kennedy (1979) defines a robust estimator to be:⁵⁴

‘one whose desirable properties are insensitive to departures from the assumptions under which it is derived.’

In the statistics literature, robust regression estimators are labelled ‘robust’ because they are relatively insensitive to extreme observations. Robust regression estimators may still retain

⁵¹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 145-180.

⁵² ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

⁵³ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 163-180.

⁵⁴ Kennedy, P., *A guide to econometrics*, MIT Press, 1979, page 24.

this desirable characteristic even when the assumptions under which they are derived do not hold. Departures from the assumptions under which robust regression estimators are derived, however, can lead to the estimators losing other important properties. In particular, departures from the assumptions under which robust regression estimators are derived can lead to the estimators losing the property of consistency. An estimator is said to be consistent if it converges in probability to the correct population value as the sample size grows. As Imbens and Wooldridge (2007) note:⁵⁵

‘so-called “robust” estimators, which are intended to be insensitive to outliers or influential data, usually require symmetry of the error distribution for consistent estimation. Thus, they are not “robust” in the sense of delivering consistency under a wide range of assumptions.’

The ERA in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013 provides an analysis of the LAD estimator.⁵⁶ As the ERA notes, the LAD estimators of the parameters of a regression will be maximum likelihood if the disturbance from the regression follows a Laplace distribution and so under this condition will be consistent. If, however, the disturbance from the regression does not follow a Laplace distribution – which is a symmetric distribution – the estimators need not be consistent. In particular, if the distribution of the disturbance is skewed, then the LAD estimators can be biased. In contrast, as Wooldridge notes:⁵⁷

‘OLS produces unbiased and consistent estimators ... whether or not the error distribution is symmetric; symmetry does not appear among the Gauss-Markov assumptions.’

It is also true that, contrary to the assertion that the ERA (2013) makes, the Gauss-Markhov Theorem does not require the disturbance from a regression to be normally distributed.^{58,59} We demonstrate using simulations that LAD estimators can be biased while OLS estimators are simultaneously unbiased when the distribution of the disturbance from a regression is skewed. We also use graphs to provide some intuition on how the bias that can be associated with the estimators can arise.

Screening strategy

The ERA, in its *Appendices to the Explanatory Statement for the Rate of Return Guidelines* of December 2013, suggests that one choose between least absolute deviations (LAD) and

⁵⁵ Imbens, G.M. and J. Wooldridge, *Quantile methods*, NBER lecture notes, 2007, page 2.

Available at:

http://www.nber.org/WNE/lect_14_quantile.pdf

⁵⁶ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 145-180.

⁵⁷ Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 333.

⁵⁸ The Gauss-Markhov Theorem states that under certain conditions OLS estimators will be Best Linear Unbiased, that is, they will have the smallest variance amongst all linear unbiased estimators.

⁵⁹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, page 166.

Wooldridge, J., *Introductory econometrics: A modern approach*, South-Western CENGAGE Learning, 2013, page 102.

OLS estimators on the basis of goodness-of-fit tests.⁶⁰ The ERA in the appendices also provides the results of bootstrap simulations that examine the behaviour of OLS and robust regression estimates.⁶¹ Surprisingly, the regulator does not take the opportunity of assessing a goodness-of-fit screening strategy like that it suggests one use.

We conduct bootstrap simulations like those that the ERA uses. We find that:

- the three robust regression techniques that the ERA employs typically provide biased estimates of beta whereas the OLS estimates exhibit no significant bias;
- the three robust regression techniques typically provide estimates of beta that are more precise;
- the tests that Puig and Stephens (2000) advocate one use can fail to detect departures from the Laplace null hypothesis;⁶² and
- a screening strategy of using a Laplace goodness-of-fit test to determine whether to use LAD does not perform well – for four of the six stocks that we use and for one of the two portfolios that we employ OLS estimates display both less bias and a lower variability than estimates that use the screening strategy.

Robust regression usage

If the benefits of using robust regression techniques exceed the costs and the market for academic research is efficient, then one should expect to find evidence of the frequent use of these techniques in published work.

We conduct keyword searches of the four major finance journals as a way of discovering how frequently robust regression techniques are used in high quality research relative to OLS.

The four journals that we select are the Journal of Finance, the Journal of Financial Economics, the Journal of Financial and Quantitative Analysis and the Review of Financial Studies. These are the four finance journals included in the list of 45 journals used by the Financial Times in compiling its business school research rankings.⁶³ They are also the four journals which a recent study of finance journal rankings that Currie and Pandher (2010) conduct rate most highly in terms of their quality.⁶⁴

We find that in these four journals there are relatively few references to robust regression techniques. We search for references to the phrase ‘ordinary least squares’ and references to nine phrases that concern three different robust regression techniques. Across the Journal of

⁶⁰ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 159-162.

⁶¹ ERA, *Appendices to the Explanatory Statement for the Rate of Return Guidelines*, December 2013, pages 163-180.

⁶² Puig, P. and M.A. Stephens, *Tests of fit for the Laplace distribution with applications*, Technometrics, 2000, pages 417-424.

⁶³ <http://www.ft.com/intl/cms/s/2/3405a512-5cbb-11e1-8f1f-00144feabdc0.html#axzz3LBbELIWS>

⁶⁴ Currie, R.R. and G. S. Pandher, *Finance journal rankings and tiers: An active scholar assessment methodology*, Journal of Banking and Finance, 2011, pages 7-20.

Finance, the Journal of Financial Economics and the Review of Financial Studies – the journals that Currie and Pandher rate most highly in terms of quality – we find 1,449 references to the phrase ‘ordinary least squares’ and 75 references to the nine phrases that concern robust regression techniques. In the Journal of Financial and Quantitative Analysis we find surprisingly few references to any of the 10 phrases. We find 27 references to the phrase ‘ordinary least squares’ and 8 references to the nine phrases that concern robust regression techniques.

Thus the evidence that we provide strongly suggests that robust regression techniques are used infrequently in high quality finance research.

Appendix A. Terms of Reference

TERMS OF REFERENCE – Robust regression techniques

The ERA has favoured the use of “robust” regression techniques, such as Least Absolute Deviation, MM and Theil-Sen. We are not aware of widespread use of these methods in the economics or finance literature, and we are aware of consultant reports which have shown that these bias the estimates of beta downwards.

Part of the motivation for the use of these methods is contained in Appendix 22, which looks at the distribution of regression errors for OLS estimates of beta. The ERA finds that these errors follow a Laplace distribution, and thus that the LAD (though not necessarily the other robust techniques) have merit. However, as is clear in the references which the ERA itself cites explaining the use of robust techniques, if datasets contain outliers (usually a key reason for using these methods), then those outliers impact not only the errors associated with the particular outlying observations, but all of the errors associated with that particular linear regression. This is because the outlier “pulls” the whole regression line away from its “true” orientation. In practical terms, it is not apparent whether looking at the errors from an OLS regression in the way the ERA has done will provide appropriate conclusions as to whether the underlying data have a distribution which favours a particular kind of “robust” regression technique.

At the same time in the recent GGT submission for its access arrangement, SFG has undertaken work to assess an appropriate equity beta which draws upon Merton’s option pricing work and the fact that debt and equity are contingent claims on the same underlying asset. SFG’s framework requires it to assess returns on debt and default probabilities in different “states of the world”, and it calculates the probabilities associated with such states.

We are aware of work within the finance literature which posits “mixing” of normal distributions. That is, a returns series might evince fat tails (or a Laplace distribution), but this might be because it is in fact comprised of two separate distributions of returns; say one during “good times” and one during “crises”. These two (or more) distributions might themselves be normal, but produce results which appear non-normal if returns are observed in an unconditional framework that does not take into account the state of the world appropriate to a particular return observation, nor the mixing proportions between the two (or more) states of the world.

For this consultancy project, we require three tasks.

The first of these is a literature review outlining the extent to which robust regression techniques are used in the finance and economics literature and, in particular, if they are not widely used, why this is the case.⁶⁵ If it is simply a case of ignorance amongst relevant professionals, then the review should say so, but if there are more fundamental reasons, then this should be highlighted. We would like the focus to be on peer-reviewed empirical work; so whilst reference can be made to previous consultant reports which have looked at the

⁶⁵ For the extent of its use, some form of citation search, or key words search of relevant databases would be sufficient.

empirical bias of these measures, they should not be the main focus. It would be useful, also, if consultants used the ERA's criteria for using regulatory discretion (see the ERA's Rate of Return Guidelines Explanatory Statement, p10) to assess these methods, as the NGL and NGR are probably too broad.

The second is a re-examination of Appendix 22 from the ERA's Rate of Return Guidelines in light of our comments above. That is, if it is incorrect to assess the structure of data by the errors from an OLS regression (as we have outlined we believe is the case above), then consultants should outline what the correct way to assess such data is, and then apply that method to the data the ERA have used. Conclusions should then be drawn as to whether the Appendix 22 has reached the correct conclusions or not.

The third task is to look more deeply at the reasons why the relevant financial data may have a particular structure, and thus whether the ERA's conclusions about the correct course of action to take in response to the data structure is itself correct. That is, if the data do not follow a normal distribution is the correct response to use robust regression techniques? Alternatively, does the nature of the structure of the data warrant a different response.

The third task should consider the SFG work referenced above. That is, is it possible to show that the structure of the data is a result of some mixing of distributions of states of the world, and can this be tied in any way to the empirical findings of SFG in its report for GGT?

The timeframe for this work is relatively tight; roughly two months. For this reason, we are not expecting the third task to be much more than exploratory at this stage. It should, however, be supported by a little more than just conjecture. In this context, there may be opportunities for further work next year, if initial conclusions are promising.

Consultants should provide a brief methodology outlining how they propose to meet the above requirements for this consultancy task. They should also outline their familiarity with the relevant literature. Consultants should also indicate whether or not they have access to the relevant data from Appendix 22, as this will be crucial. DBP is required to submit its access arrangement to the ERA in December, so timing is crucial; a draft report with substantive conclusions should be complete by the end of October. This is particularly important in respect of scoping the third task above. The quotation should also be a fixed fee, for the tasks outlined above, however, consultants should note that this may be an iterative process to getting the report done – ie:

- Preparing an initial draft of each report
- Getting each draft reviewed by DBP's lawyers (including counsel)
- Perhaps a conference with DBP's lawyers to address areas of refinement/addition to the draft report
- Issuing a penultimate version of each report for final review by the client and assessing relevant extracts of DBP's rate of return submission/s to which each report relates
- Issuing a final version of each report.

Consultants should note that the scope may also be extended to request the consultant to review any draft decision that the ERA may release in relation to the ATCO and GGT access arrangement revisions. Interaction with DBP's legal counsel and any work involving review

of the ATCO and GGT decisions can be additional to the fixed fee, and payable on an hourly basis. We would therefore require an indication of hourly charge-out rates.

As a final point, consultants should be aware that it is likely that aspects of the work may be included in a challenge before the Australian Competition Tribunal if the ERA disagrees with our findings. As such, the work must be undertaken to the standards required by the Federal Court's *Expert Witness Guidelines* (attached), and reports should contain the following disclaimers:

"I declare that I have made all the inquiries that I believe are desirable and appropriate and that no matters of significance that I regard as relevant have, to my knowledge, been withheld from the Court."

"I understand that my duty is to the Court and not to those who have retained my services in this matter. I have been given and have read, understood and complied with Practice Note CM7 issued by the Federal Court of Australia concerning guidelines for expert witnesses."

"I am not aware of any actual or perceived conflict of interest which would compromise my independence in this matter."

Appendix B. Federal Court Guidelines

FEDERAL COURT OF AUSTRALIA

Practice Note CM 7

EXPERT WITNESSES IN PROCEEDINGS IN THE FEDERAL COURT OF AUSTRALIA

Practice Note CM 7 issued on 1 August 2011 is revoked with effect from midnight on 3 June 2013 and the following Practice Note is substituted.

Commencement

1. This Practice Note commences on 4 June 2013.

Introduction

2. Rule 23.12 of the Federal Court Rules 2011 requires a party to give a copy of the following guidelines to any witness they propose to retain for the purpose of preparing a report or giving evidence in a proceeding as to an opinion held by the witness that is wholly or substantially based on the specialised knowledge of the witness (see **Part 3.3 - Opinion** of the *Evidence Act 1995* (Cth)).
3. The guidelines are not intended to address all aspects of an expert witness's duties, but are intended to facilitate the admission of opinion evidence⁶⁶, and to assist experts to understand in general terms what the Court expects of them. Additionally, it is hoped that the guidelines will assist individual expert witnesses to avoid the criticism that is sometimes made (whether rightly or wrongly) that expert witnesses lack objectivity, or have coloured their evidence in favour of the party calling them.

Guidelines

1. General Duty to the Court⁶⁷

- 1.1 An expert witness has an overriding duty to assist the Court on matters relevant to the expert's area of expertise.
- 1.2 An expert witness is not an advocate for a party even when giving testimony that is necessarily evaluative rather than inferential.
- 1.3 An expert witness's paramount duty is to the Court and not to the person retaining the expert.

⁶⁶ As to the distinction between expert opinion evidence and expert assistance see *Evans Deakin Pty Ltd v Sebel Furniture Ltd* [2003] FCA 171 per Allsop J at [676].

⁶⁷ The "*Ikarian Reefer*" (1993) 20 FSR 563 at 565-566.

2. The Form of the Expert's Report⁶⁸

- 2.1 An expert's written report must comply with Rule 23.13 and therefore must
 - (a) be signed by the expert who prepared the report; and
 - (b) contain an acknowledgement at the beginning of the report that the expert has read, understood and complied with the Practice Note; and
 - (c) contain particulars of the training, study or experience by which the expert has acquired specialised knowledge; and
 - (d) identify the questions that the expert was asked to address; and
 - (e) set out separately each of the factual findings or assumptions on which the expert's opinion is based; and
 - (f) set out separately from the factual findings or assumptions each of the expert's opinions; and
 - (g) set out the reasons for each of the expert's opinions; and
 - (ga) contain an acknowledgment that the expert's opinions are based wholly or substantially on the specialised knowledge mentioned in paragraph (c) above⁶⁹; and
 - (h) comply with the Practice Note.
- 2.2 At the end of the report the expert should declare that "[the expert] has *made all the inquiries that [the expert] believes are desirable and appropriate and that no matters of significance that [the expert] regards as relevant have, to [the expert's] knowledge, been withheld from the Court.*"
- 2.3 There should be included in or attached to the report the documents and other materials that the expert has been instructed to consider.
- 2.4 If, after exchange of reports or at any other stage, an expert witness changes the expert's opinion, having read another expert's report or for any other reason, the change should be communicated as soon as practicable (through the party's lawyers) to each party to whom the expert witness's report has been provided and, when appropriate, to the Court⁷⁰.
- 2.5 If an expert's opinion is not fully researched because the expert considers that insufficient data are available, or for any other reason, this must be stated with an indication that the opinion is no more than a provisional one. Where an expert witness who has prepared a report believes that it may be incomplete or inaccurate without some qualification, that qualification must be stated in the report.
- 2.6 The expert should make it clear if a particular question or issue falls outside the relevant field of expertise.

⁶⁸ Rule 23.13.

⁶⁹ See also *Dasreef Pty Limited v Nawaf Hawchar* [2011] HCA 21.

⁷⁰ The "*Ikarian Reefer*" [1993] 20 FSR 563 at 565

- 2.7 Where an expert's report refers to photographs, plans, calculations, analyses, measurements, survey reports or other extrinsic matter, these must be provided to the opposite party at the same time as the exchange of reports⁷¹.

3. Experts' Conference

- 3.1 If experts retained by the parties meet at the direction of the Court, it would be improper for an expert to be given, or to accept, instructions not to reach agreement. If, at a meeting directed by the Court, the experts cannot reach agreement about matters of expert opinion, they should specify their reasons for being unable to do so.

J L B ALLSOP

Chief Justice

4 June 2013

⁷¹ The "*Ikarian Reefer*" [1993] 20 FSR 563 at 565-566. See also Ormrod "*Scientific Evidence in Court*" [1968] Crim LR 240

Appendix C. Curriculum Vitae

Simon M. Wheatley

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 Blackburn VIC 3130
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Overview

Simon is a consultant and was until 2008 a Professor of Finance at the University of Melbourne. Since 2008, Simon has applied his finance expertise in investment management and consulting outside the university sector. Simon's interests and expertise are in individual portfolio choice theory, testing asset-pricing models and determining the extent to which returns are predictable. Prior to joining the University of Melbourne, Simon taught finance at the Universities of British Columbia, Chicago, New South Wales, Rochester and Washington.

Personal

Nationalities:	U.K. and U.S.
Permanent residency:	Australia

Employment

- Affiliated Industry Expert, NERA Economic Consulting, 2014-
- Special Consultant, NERA Economic Consulting, 2009-2014
- External Consultant, NERA Economic Consulting, 2008-2009
- Quantitative Analyst, Victorian Funds Management Corporation, 2008-2009
- Adjunct, Melbourne Business School, 2008
- Professor, Department of Finance, University of Melbourne, 2001-2008
- Associate Professor, Department of Finance, University of Melbourne, 1999-2001
- Associate Professor, Australian Graduate School of Management, 1994-1999
- Visiting Assistant Professor, Graduate School of Business, University of Chicago, 1993-1994
- Visiting Assistant Professor, Faculty of Commerce, University of British Columbia, 1986

- Assistant Professor, Graduate School of Business, University of Washington, 1984-1993
- **Education**
- Ph.D., University of Rochester, USA, 1986; Major area: Finance; Minor area: Applied statistics; Thesis topic: Some tests of international equity market integration; Dissertation committee: Charles I. Plosser (chairman), Peter Garber, Clifford W. Smith, Rene M. Stulz
- M.A., Economics, Simon Fraser University, Canada, 1979
- M.A., Economics, Aberdeen University, Scotland, 1977

Publicly Available Reports

Imputation Credits and Equity Returns: A report for the Energy Networks Association, October 2013, <http://www.aer.gov.au/sites/default/files/ENA,%20Attachment%204%20-%20NERA%20Report%20-%20Imputation%20Credits%20and%20Equity%20Prices,%20Submission%20to%20draft%20AER%20rate%20of%20return%20guideline%20-%202011%20Oct%202013.pdf>

The Fama-French Three-Factor Model: A report for the Energy Networks Association, October 2013, http://www.aer.gov.au/sites/default/files/Essential%20Energy%20-%20Attachment%207.9_NERA_The%20Fama-French%20Three-Factor%20Model%20-%202014.pdf

The Market Risk Premium: Analysis in Response to the AER's Draft Rate of Return Guidelines: A report for the Energy Networks Association, October 2013, <http://www.aer.gov.au/sites/default/files/ENA,%20Attachment%203%20-%20NERA%20Report%20-%20The%20Market%20Risk%20Premium,%20Submission%20to%20draft%20AER%20Rate%20of%20return%20guideline%20-%202011%20Oct%202013.pdf>

The Market, Size and Value Premiums: A report for the Energy Networks Association, June 2013, <http://www.aer.gov.au/sites/default/files/Report%2015%20-%20ENAMRPRReport28062013%20Final.pdf>

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The Payout Ratio: A report for the Energy Networks Association, June 2013, [http://www.aer.gov.au/sites/default/files/Report%2012%20-%20Payout%20Ratio%20\(Final\)%20-%20June%202013.pdf](http://www.aer.gov.au/sites/default/files/Report%2012%20-%20Payout%20Ratio%20(Final)%20-%20June%202013.pdf)

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Consulting Experience

NERA, 2008-present

Lumina Foundation, Indianapolis, 2009

Industry Funds Management, 2010

Academic Publications

Imputation credits and equity returns, (with Paul Lajbcygier), 2012, *Economic Record* 88, 476-494.

Do measures of investor sentiment predict returns? (with Robert Neal), 1998, *Journal of Financial and Quantitative Analysis* 33, 523-547.

Adverse selection and bid-ask spreads: Evidence from closed-end funds (with Robert Neal), 1998, *Journal of Financial Markets* 1, 121-149.

Shifts in the interest-rate response to money announcements: What can we say about when they occur? (with V. Vance Roley), 1996, *Journal of Business and Economic Statistics* 14, 135-138.

International investment restrictions and closed-end country fund prices, (with Catherine Bonser-Neal, Gregory Brauer, and Robert Neal), 1990, *Journal of Finance* 45, 523-547 (reprinted in *International Capital Markets Volume III*, 2003, G. Andrew Karolyi and Rene M. Stulz, editors, Edward Elgar Publishing, Cheltenham, Glos).

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Some tests of international equity market integration, 1988, *Journal of Financial Economics* 21, 177-212 (reprinted in *International Capital Markets Volume I*, 2003, G. Andrew Karolyi and Rene M. Stulz, editors, Edward Elgar Publishing, Cheltenham, Glos).

Some tests of the consumption-based asset pricing model, 1988, *Journal of Monetary Economics* 22, 193-215.

Working Papers

An evaluation of some alternative models for pricing Australian stocks (with Paul Lajbcygier), 2009.

Intertemporal substitution, small-sample bias, and the behaviour of U.S. household consumption (with Kogulakrishnan Maheswaran and Robert Porter), 2007.

Keeping up with the Joneses, human capital, and the home-equity bias (with En Te Chen), 2003.

Evaluating asset pricing models, 1998.

Time-non-separable preferences or artifact of temporal aggregation? (with Robert Porter), 2002.

Testing asset pricing models with infrequently measured factors, 1989.

Refereeing Experience

Referee for Accounting and Finance, the Australian Journal of Management, Economic Letters, Financial Analysts Journal, Financial Management, Journal of Accounting and Economics, Journal of Business, Journal of Empirical Finance, Journal of Finance, Journal of Financial and Quantitative Analysis, Journal of Financial Economics, Journal of Futures Markets, Journal of International Economics, Journal of International Money and Finance, Journal of Money, Credit, and Banking, Journal of Monetary Economics, Management Science, National Science Foundation, Pacific-Basin Finance Journal, and the Review of Financial Studies.

Program Committee for the Western Finance Association in 1989 and 2000.

Teaching Experience

International Finance, Melbourne Business School, 2008

Corporate Finance, International Finance, Investments, University of Melbourne, 1999-2008

Corporate Finance, International Finance, Investments, Australian Graduate School of Management, 1994-1999

Investments, University of Chicago, 1993-1994

Investments, University of British Columbia, 1986

International Finance, Investments, University of Washington, 1984-1993

Investments, Macroeconomics, Statistics, University of Rochester, 1982

Accounting, 1981, Australian Graduate School of Management, 1981

Teaching Awards

MBA Professor of the Quarter, Summer 1991, University of Washington

Computing Skills

User of SAS since 1980. EViews, Excel, EXP, LaTeX, Matlab, Powerpoint, Visual Basic. Familiar with the Australian School of Business, Compustat and CRSP databases. Some familiarity with Bloomberg, FactSet and IRESS.

Board Membership

Anglican Funds Committee, Melbourne, 2008-2011

Honours

Elected a member of Beta Gamma Sigma, June 1986.

Fellowships

Earhart Foundation Award, 1982-1983

University of Rochester Fellowship, 1979-1984

Simon Fraser University Fellowship, 1979

Inner London Education Authority Award, 1973-1977

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